

Experiment 3

Electron Diffraction

Introduction

In 1924, de Broglie proposed that the motion of particles was described by a wave equation. He associated with each particle a wave, the de Broglie wave, whose wavelength λ depended on the momentum p of the particle according to

$$\lambda = \frac{h}{p} \quad (3.1)$$

where h is Planck's constant. In this experiment you will measure the de Broglie wave-length for fast moving electrons, and show that it is related to its momentum according to equation (3.1).

Theory

If an electron leaves a cathode with zero velocity, then its kinetic energy after traversing an accelerating potential V is given by:

$$\frac{1}{2}mv^2 = eV \quad (3.2)$$

where m is the electron mass, v is its speed after traversing the potential V , and e is its charge. The momentum p of the electron is then given by

$$p = mv = \sqrt{2meV}$$

and its de Broglie wavelength

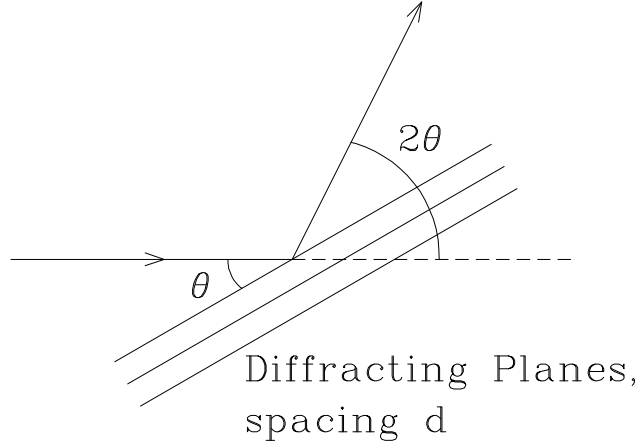
$$\lambda = \frac{h}{\sqrt{2meV}}. \quad (3.3)$$

If this electron strikes a crystal, its de Broglie wave will be diffracted by the 3-dimensional periodic structure of the crystal. The electron will pass through the crystal only in those directions in which the de Broglie wave is diffracted strongly by the crystal. These directions are given by Bragg's Law:

$$2d \sin \theta = n\lambda \quad (3.4)$$

Therefore, if we know the d -values for a crystal (and we can calculate these if the crystal structure is known), then the wavelength λ can be found by measuring the directions (2θ) in which the waves are strongly diffracted.

Consider now the situation where a parallel beam of waves is incident on many crystals, which are oriented in a completely random fashion with respect to the beam. In such a random aggregate of crystals there are always crystals with the same set of lattice planes in an orientation satisfying



Bragg's Law. All these crystals will diffract the incident beam strongly over the same angle 2θ : the result is a diffracted beam in the shape of a cone with top angle 4θ .

This is the powder method of diffraction (Cullity, Chapter 3, p.96) and in effect is the technique used in this experiment to measure the wavelength λ . Our beam of electrons is incident on a polycrystalline foil of aluminum. The cones of diffracted electrons strike a fluorescent screen at a distance D , forming a set of fluorescent rings of radius r . It is evident that diffraction angle 2θ is given by

$$\tan 2\theta = \frac{r}{D}. \quad (3.5)$$

The Bragg's Law

$$2d \sin \theta = n\lambda \quad (3.6)$$

may be rewritten as

$$2 \left(\frac{d}{n} \right) \sin \theta = \lambda \quad (3.7)$$

In equation (3.4) we refer to θ as the direction of the n_{th} order beam diffracted from the set of atomic planes with spacing d . In the alternative formulation (equation (3.6)) we may refer to θ as the direction of the 1st order beam diffracted from a set of planes with spacing d/n . For cubic crystals the distance d between successive lattice planes in the set specified by the Miller indices hkl is

$$d = \frac{a}{\sqrt{h^2 + k^2 + \ell^2}}$$

where a is the unit cell side length. Hence we may write

$$\frac{d}{n} = \frac{a}{n\sqrt{h^2 + k^2 + \ell^2}} = \frac{a}{\sqrt{H^2 + K^2 + L^2}} \quad (3.8)$$

where $H = nh, K = nk, L = n\ell$ are the Miller indices of a set of planes with spacing d/n .

In the present experiment, the diffraction angles are sufficiently small that $\tan 2\theta \cong 2\theta$ and $\sin \theta \cong \theta$. Hence, combining equations (3.6), (3.7), and (3.8) gives an expression for the electron beam wavelength in terms of the radii of the diffraction rings on the fluorescent screen, namely

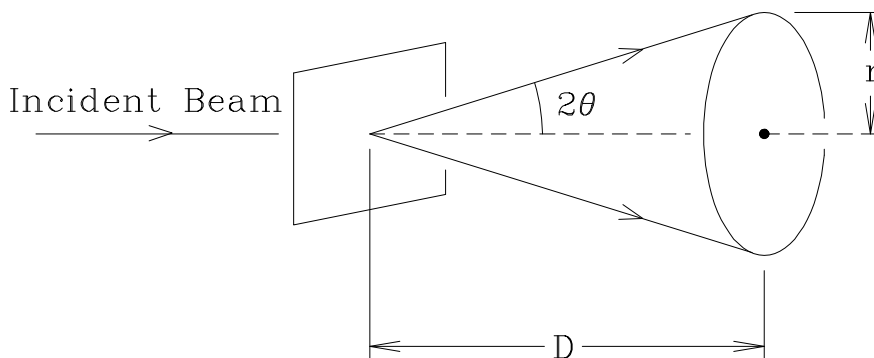
$$\lambda = \frac{a}{\sqrt{H^2 + K^2 + L^2}} \cdot \frac{r}{D}. \quad (3.9)$$

Finally, by equating the expression for the diffraction measurement of the electron beam wavelength (equation (3.9)) to the de Broglie expression for the electron beam wavelength written in terms of the electron tube accelerating voltage V (equation (3.3)) one obtains

$$r = \sqrt{H^2 + K^2 + L^2} \left(\frac{D}{a} \cdot \frac{h}{\sqrt{2me}} \right) \frac{1}{\sqrt{V}} \quad (3.10)$$

Equation (3.10) gives an explicit expression for the radii of a set of diffraction rings as a function of accelerating tube voltage and its validity rests on that of the de Broglie postulate.

Experimental



The instrument is a modified cathode-ray oscilloscope with a variable accelerating potential V . The electron beam can be deflected with horizontal and vertical deflection plates till it strikes an Al foil in the tube. The diffraction pattern of the Al foil can be measured on the screen.

Obtain a diffraction pattern at $V = 4000$ V, following the instruction sheet supplied with the instrument. Measure the diameters of the diffraction rings five times with calipers. In order to minimize the effects of ring distortion, make the five measurements on each ring in different directions. Repeat the measurements as a function of V up to $V = 9000$ V. DO NOT EXCEED $5 \mu\text{A}$ BEAM CURRENT.

From the data calculate average values of r for each ring, at each value of V . The value of D for this instrument is:

$$D = 18.2 \pm 0.2 \text{ cm.}$$

The value of a for aluminum is:

$$a = 4.05 \text{ \AA.}$$

Verify equation (3.10).