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Experiment 5

Resistivity and Hall effect of a thin Bismuth film

Introduction

The conductivity σ of a solid depends on two parameters: one is the number of charge carriers (in metals, electrons of charge $-e$) per unit volume, n. The other factor is the velocity with which the carriers will move through the solid when an electric field is applied. The ratio of this drift velocity (in ms^{-1}) to the applied field (in Vm^{-1}) is called the mobility μ (in $\mathrm{m}^2\mathrm{V}^{-1}\mathrm{s}^{-1}$). The conductivity is given by

$$\sigma = en\mu \tag{5.1}$$

The conductivity (or its inverse, the resistivity ρ) can in principle be determined by measuring the resistance of a block of length ℓ and cross-section area $A(= wxh)$.

If a current i is passed through the block, and a potential difference V is measured at two points ℓ apart then

$$\rho = \frac{V}{i} \times \frac{wxh}{\ell} \tag{5.2}$$

In practice one rarely has (or can make) a sample in this ideal shape, and a number of techniques are available to determine ρ from $i - V$ measurements on samples of irregular shape. We will use a method developed by van der Pauw, for homogeneous samples of constant thickness d, but otherwise of arbitrary shape.

Four contacts, labelled A, B, C and D, are attached to the side of the flat sample in arbitrary positions. A current i_{AB} is passed from A to B, and the resulting voltage difference V_{CD} is measured between contacts C and D; call the ratio V_{CD}/i_{AB} a "resistance";

$$R_{AB,CD} = R_1.$$

Next, the current is passed from B to C, and the voltage V_{DA} is measured:

$$R_{BC,DA} = R_2.$$

Similarly, determine V_{AB}/i_{CD}, which equals R_1, and V_{BC}/i_{DA} which equals R_2 again. Now van der Pauw showed that R_1 and R_2 are related to the sample resistivity ρ and thickness d by:

$$\exp\left(-\frac{\pi d R_1}{\rho}\right) + \exp\left(-\frac{\pi d R_2}{\rho}\right) = 1 \tag{5.3}$$

Equation (5.3) cannot be solved for ρ in closed form, but it can be rewritten as:

$$\rho = \frac{\pi d}{\ln 2} \cdot \frac{(R_1 + R_2)}{2} f\left(\frac{R_1}{R_2}\right) \tag{5.4}$$

where the van der Pauw function f of the argument R_1/R_2 satisfies

$$\cosh\left[\frac{\ln 2}{f} \cdot \frac{(R_1/R_2) - 1}{(R_1/R_2) + 1)}\right] = \frac{1}{2} \exp\left[\frac{\ln 2}{f}\right] \tag{5.5}$$

Values of $f(R_1/R_2)$ have been tabulated; therefore after measuring R_1 and R_2 and calculating R_1/R_2 for the sample its resistivity ρ can be calculated from equation (5.4). Notice that the van der Pauw method requires the measurement of only one dimension (d) of the sample; the standard method requires three measurements of sample dimensions.

The Hall effect is the sideways deflection of the moving charge carriers in the sample due to a magnetic field $\vec{B}$ perpendicular to the current $\vec{i}$. The electron flowing in the (negative) x-direction experiences a Lorentz force $q\vec{V} \times \vec{B}$ in the (negative) z-direction due to a $\vec{B}$ field in the (positive) y-direction; the result is a potential difference, the Hall voltage V_H, developing between the top and bottom face of the sample, (positive) z-direction. The Hall coefficient R_H is now defined as:

$$RH = \frac{V_{PQ} \cdot w}{i \cdot B} \tag{5.6}$$

It is related to the carrier concentration n from equation (5.1) by

$$n = \frac{1}{eR_H} \tag{5.7}$$

Once n is known, the mobility μ of the charge carriers can be calculated (from equation (5.7)):

$$\mu = \frac{R_H}{\rho} \tag{5.8}$$

The van der Pauw method allows an easy determination of R_H for a flat sample: Pass a current i_{AC} between opposite contacts; it gives a voltage V_{BD}. Now put a $\vec{B}$ field perpendicular to the sample; V_{BD} changes to a new value $V_{BD} + \Delta V_H$. The Hall coefficient is related to ΔV_H by

$$R_H = \frac{\Delta V_H \cdot d}{i_{AC} \cdot B} \tag{5.9}$$

Procedure

To determine ρ, n and μ for a thin Bismuth film:

1. Place the film in the sample holder and attach the four contacts A, B, C and D to the film.

2. Connect a circuit from a current source, a voltmeter and ammeter to measure i_{AB} and V_{CD}. Do the measurements, and calculate R_1. Reverse the direction of the current leads and determine R_1 again. The two values of R_1 should be identical unless you have bad(rectifying) contacts.

3. Similarly determine $R_2 = I_{BC}/V_{AD}$; also check that indeed R_1 is also equal to i_{CD}/V_{AB} and R_2 to i_{AD}/V_{BC}. Calculate average values of R_1, R_2 and R_2/R_1.

4. From the graph of the van der Pauw function $f(R_1/R_2)$ determine the appropriate f value for your sample, and calculate ρ.

5. Place the samples between the poles of the electromagnet, and measure i_{AC} and V_{BD}. Switch the $\vec{B}$-field on, and measure the Hall voltage ΔV_H. Reconnect the leads to measure i_{BD}, and V_{AC}, and measure ΔV_H again. Reverse the current leads to check for non-Ohmic contacts. Reverse the direction of the $\vec{B}$-field, and check that ΔV_H changes sign.

6. Calculate R_H from the average ΔV_H value, and calculate n and μ for Bismuth. Compare n (the number of "free" electrons per unit volume) to the number of Bi atoms per unit volume.

References

1. D. Halliday and R. Resnick, *Fundamentals of Physics*, Chapters 28, 30-4

2. K. Seeger, *Semiconductor Physics*, Chapter 4.3

3. E. H. Putley, *The Hall Effect and Semi-conductor Physics*, Chapters 1-2 and 2-1.