Physics 4P41

Homework assignment No. 1

Due January 31, 2019 (Thu)

Questions **Marks**

1. Langevin paramagnet is a system of N non-interacting classical magnetic dipoles described by the Hamiltonian

$$H = \sum_{i=1}^{N} \frac{\mathbf{p}_i^2}{2M} - \sum_{i=1}^{N} \mathbf{m}_i \mathbf{B},$$

where $\mathbf{m}_i$ are the magnetic dipole moments of the particles ($|\mathbf{m}_i| = m$), and $\mathbf{B}$ is the external magnetic field. Assuming that $\mathbf{B} \parallel \hat{z}$ in 3D,

(a) calculate the expectation value of the z-component of the magnetic moment per particle at arbitrary B, and the susceptibility $\chi(T)$ at small B. **(2)**

(b) calculate the internal energy and the Helmholtz free energy per particle as functions of temperature and magnetic field (express your answer in terms of $x = mB/k_BT$); **(2)**

(c) calculate and plot the specific heat per particle c_V as a function of $y = 1/x = k_BT/mB$. What are the asymptotics of c_V at $T \to 0$ and $T \to \infty$? **(2)**

2. Do 1(a,b,c) for a system of N fixed non-interacting quantum magnetic dipoles (spins 1/2) described by the Hamiltonian

$$H = -B \sum_{i=1}^{N} m_z,$$

where $m_z = \pm m$ is the spin magnetic moment.

(a) **(1)**

(b) **(1)**

(c) **(1)**

3. The energy of a 1D classical anharmonic oscillator is given by $H = p^2/2m + bx^{2n}$, where $n > 1$. Consider a thermodynamic system composed of a large number N of the identical non-interacting oscillators, at constant T and V.
   Calculate the specific heat per oscillator c_V. **(2)**

4. For a ultra-relativistic gas of non-interacting classical particles in 3D with $E(\mathbf{p}) = c|\mathbf{p}|$, find

   (a) the internal energy and the Helmholtz free energy per particle as functions of T; **(1)**

   (b) the equation of state (i.e. a relation between p, V, N, and T). **(1)**

5. For an ideal classical gas of monoatomic molecules of mass m in 3D, calculate the average fluctuation of the total energy:

$$\frac{\sigma_E}{\langle E \rangle} = \frac{\sqrt{\langle E^2 \rangle - \langle E \rangle^2}}{\langle E \rangle}.$$

   Assume that the temperature T, volume V, and the number of particles N are kept constant. **(2)**

Total = 15