Physics 4P41

Homework assignment No. 2

Due February 25, 2019 (Mon)

Questions **Marks**

1. A small subsystem of a very large container filled with an ideal gas of identical particles is regarded as a grand canonical ensemble. Show that the probability of finding n atoms in the subsystem is given by the Poisson distribution

$$p(n) = \frac{\langle n \rangle^n}{n!} e^{-\langle n \rangle},$$

where $\langle n \rangle$ is the average number of atoms in the subsystem. **(2)**

2. Let the density of states of the electrons in some solid be a constant, D, for $\epsilon > 0$, and zero for $\epsilon < 0$. The concentration of electrons: $N/V = n$.

(a) Calculate the chemical potential $\mu(T)$ at $T > 0$. What is the Fermi energy ϵ_F? **(1)**

(b) Calculate the specific heat per particle at $T \ll T_F$. Express your answer in terms of T/T_F. **(2)**

(c) Calculate the entropy per particle at $T \ll T_F$. Express your answer in terms of T/T_F. **(2)**

3. Consider a degenerate 2D ideal gas of N electrons (spin 1/2) placed in an external magnetic field $B = B\hat{z}$:

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2m} - \sum_{i=1}^{N} B m_{i,z},$$

where the spin magnetic moment is $m_{i,z} = \mu_B$ for spin $+1/2$, and $m_{i,z} = -\mu_B$ for spin $-1/2$. Calculate the spin magnetic susceptibility per unit volume

$$\chi(T) = \frac{\partial(M/V)}{\partial B}$$

at $T = 0$.
Hint: Magnetization of the system is defined as $\mu_B(N_\uparrow - N_\downarrow)$, which should be calculated in the limit of $\mu_B B \to 0$. **(2)**

4. Find the pressure as a function of T and concentration $n = N/V$ in a 3D non-relativistic Fermi-gas ($\epsilon = \mathbf{p}^2/2m$) at $T \ll T_F$. Plot the isotherm of this Fermi-gas on the pv-plane, where $v = n^{-1}$ is the volume per particle. **(2)**

Total = 11