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The scattering of slow neutrons by *ortho*- and *para*-hydrogen

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The neutron velocity selector of the Cavendish Laboratory has been used to measure the scattering cross-sections of *ortho*- and *para*-hydrogen for slow neutrons. The triplet and singlet scattering amplitudes of the neutron-proton interaction may be deduced from these cross-sections. The values obtained are

$$a_t = (0.537 \pm 0.004) \times 10^{-12} \text{ cm},$$

$$a_s = -(2.373 \pm 0.007) \times 10^{-12} \text{ cm},$$

where a_t and a_s are the triplet and singlet scattering amplitudes respectively.

The values of the coherent scattering amplitude $f = 2(\frac{3}{4}a_t + \frac{1}{4}a_s)$, and of the free proton cross-section $\sigma_f = 4\pi(\frac{3}{4}a_t^2 + \frac{1}{4}a_s^2)$ given by the above values of a_t and a_s , are

$$f = -(0.380 \pm 0.005) \times 10^{-12} \text{ cm},$$

$$\sigma_f = (20.41 \pm 0.14) \times 10^{-24} \text{ cm}^2.$$

1. INTRODUCTION

When a beam of slow neutrons is scattered by a nucleus, only the states of zero orbital angular momentum, i.e. the *S* states, are disturbed. The scattered wave is therefore spherically symmetrical, and if the incident wave is represented by e^{ikz} the scattered wave may be represented by $-(a/r)e^{ikr}$. k is the wave number of the neutrons. The quantity a is known as the scattering amplitude. When the scattering nucleus is a proton, the total spin of the neutron-proton system is either one (triplet state) or zero (singlet state). The scattering amplitudes for the two states, denoted by a_t and a_s respectively, are fundamental parameters in the phenomenological theory of the neutron-proton interaction.

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It was first pointed out by Schwinger & Teller (1937) that measurement of the scattering cross-sections of *ortho*- and *para*-hydrogen for slow neutrons is one method of determining these two scattering amplitudes. In a molecule of *ortho*-hydrogen the spins of the two protons are parallel. Thus the two scattered waves from such a molecule are either both due to a total neutron-proton spin of one or both due to a total spin of zero. In *para*-hydrogen, on the other hand, the two proton spins are anti-parallel, and one of the scattered neutron waves corresponds to a neutron-proton spin of one and the other to a spin of zero. The two scattered waves from a hydrogen molecule interfere, and, from the observed scattering cross-sections of *ortho*- and *para*-hydrogen, the triplet and singlet scattering amplitudes may be deduced.

The first measurements of the *ortho*- and *para*-hydrogen cross-sections were made by Halpern, Estermann, Simpson & Stern (1937) and Brickwedde, Dunning, Hoge & Manley (1938). Their results showed that the cross-section of *ortho*- was several times larger than that of *para*-hydrogen, from which it was possible to deduce that the singlet amplitude was negative. However, their scattering specimens were in the liquid state and the effects due to the intermolecular forces in the liquid were difficult to interpret. Subsequently, Alvarez & Pitzer (1940) and Sutton *et al.* (1947) made measurements with gaseous specimens. The results of the former experiment were in disagreement with the results of other methods of measuring the scattering amplitudes. The results of the latter were in closer agreement with the results of other methods, but nevertheless some discrepancy still existed. It appeared possible that this was due to an uncertainty in the *ortho-para* composition of the scattering gas. For this reason and because of the importance of the scattering amplitudes in the theory of the neutron-proton force, it was decided to repeat the experiment once more. A brief report of the work has already been published (Stewart & Squires 1953).

2. THEORY

(a) *Ortho- and para-hydrogen*

To a first approximation a hydrogen molecule may be regarded as a rigid rotator. Its rotational energy is given by

$$E_J = \frac{\hbar^2}{2I} J(J+1),$$

where I is the moment of inertia of the molecule about an axis perpendicular to the line joining the nuclei, and J is an integer. Because of the symmetry properties of the wave function, an *ortho* molecule can only possess odd values of J and a *para* molecule can only possess even values of J .

In an equilibrium mixture of *ortho*- and *para*-hydrogen at temperature T , the number of molecules with energy E_J is proportional to $p_J \exp(-E_J/kT)$, where p_J , the statistical weight of the energy level, is given by

$$\begin{aligned} p_J &= 2J+1, & J \text{ even,} \\ p_J &= 3(2J+1), & J \text{ odd.} \end{aligned}$$

k is Boltzmann's constant.

In the present paper the *ortho*-*para* composition of a mixture of the two modifications will be specified by the quantity f_p , the fraction of *para* molecules. At high temperatures ($kT \gg \hbar^2/2I$), the equilibrium value of f_p tends to 0.25. This limiting mixture is known as normal hydrogen. The equilibrium mixture at room temperature (290° K) is almost normal, the fraction of *para* molecules being 0.2507. Almost pure *para*-hydrogen may be prepared by cooling normal hydrogen to a low temperature ($\sim 20^\circ$ K) in the presence of charcoal. The equilibrium mixture is obtained in a short time.

(b) *Scattering of slow neutrons by ortho- and para-hydrogen*

Collisions between a neutron and a hydrogen molecule may be classified according to the initial and final J values of the molecule. Denote the cross-section for a collision in which the rotational quantum number changes from J to J' by $\sigma_{JJ'}$.

The theory of the scattering of slow neutrons by *ortho*- and *para*-hydrogen has been given by Schwinger & Teller (1937). They derive expressions for the total scattering cross-sections in the following form:

$$\left. \begin{aligned} \sigma_{00} &= k_{00}(3a_t + a_s)^2, \\ \sigma_{01} &= k_{01}(a_t - a_s)^2, \\ \sigma_{11} &= k_{11}\{(3a_t + a_s)^2 + 2(a_t - a_s)^2\}, \\ \sigma_{10} &= k_{10}(a_t - a_s)^2. \end{aligned} \right\} \quad (1)$$

The coefficients $k_{JJ'}$ are given as functions of $\xi = 2\pi r_e/\lambda$, where r_e = equilibrium distance between the protons and λ = reduced wave-length of the neutron $= \frac{3}{2}(\hbar/Mv_r)$. M = mass of neutron, v_r = relative velocity of neutron and hydrogen molecule.

Owing to the thermal motion of the gas molecules, the cross-section must be averaged over all possible values of v_r to obtain σ' , the effective cross-section. Denote the velocity of the neutrons by v_n and the temperature of the scattering gas by T . Then σ' is given by

$$\sigma' = \sigma'(v_n, T) = \frac{1}{v_n} \int_{v_m=0}^{v_m=\infty} v_r \sigma(v_r) M(v_m) dv_m, \quad (2)$$

where v_m is the velocity of a gas molecule and $M(v_m)$ is the Maxwellian distribution function for the gas at temperature T . By analogy with (1) we write

$$\sigma'_{00} = k'_{00}(3a_t + a_s)^2,$$

and so on for σ'_{01} , σ'_{11} and σ'_{10} .

(c) *Choice of velocity range of incident neutrons and temperature of hydrogen gas*

The temperature of the scattering gas was 20° K, the boiling-point of liquid hydrogen. At this temperature the fraction of *para*-hydrogen molecules not in the $J = 0$ state is 1 in 10^{11} , while the fraction of *ortho*-hydrogen molecules not in the

$J = 1$ state is 1 in 10^{18} . The effective scattering cross-sections for *para*- and *ortho*-hydrogen are therefore

$$\sigma'_{para} = \sigma'_{00} + \sigma'_{01} = k'_{00}(3a_t + a_s)^2 + k'_{01}(a_t - a_s)^2, \quad (3)$$

$$\sigma'_{ortho} = \sigma'_{11} + \sigma'_{10} = (2k'_{11} + k'_{10})(a_t - a_s)^2 + k'_{11}(3a_t + a_s)^2. \quad (4)$$

The numerical values of a_t and a_s are such that $(a_t - a_s)^2$ is about 15 times $(3a_t + a_s)^2$. The $(a_t - a_s)^2$ term accounts for about 97 % of the *ortho* cross-section, which means that a relatively inaccurate value of $(3a_t + a_s)^2$ suffices to obtain $(a_t - a_s)^2$ from this cross-section.

The quantity $(3a_t + a_s)^2$ may be deduced from the *para* cross-section, but it is essential that the cross-section σ'_{01} be kept small. Otherwise, the measurement of σ'_{para} , like that of σ'_{ortho} , simply gives $(a_t - a_s)^2$. The cross-section σ_{01} is zero for collisions in which the kinetic energy E of the neutron-molecule system about its centre of mass is less than the energy difference E_1 between the $J = 0$ and $J = 1$ states. By maintaining the hydrogen at a low temperature and using very slow neutrons, we can make the proportion of collisions in which $E > E_1$ small, and hence ensure that σ'_{01} is small.

The requirement of very slow neutrons conflicts with the need for a high counting rate, because as the velocity of the neutrons selected is decreased the number of neutrons available decreases also. The fastest neutrons selected for the measurement of σ'_{para} had an inverse velocity of $700 \mu\text{s/m}$. For such neutrons and with the hydrogen kept at 20°K , σ'_{01} amounts to about 10 % of σ'_{para} . (Cross-sections of the type σ'_{0n} and σ'_{1n} ($n \geq 2$) have been ignored in equations (3) and (4) and in the subsequent discussion. They are negligible at the neutron energies and gas temperatures under consideration.)

(d) Computation of the coefficients $k_{JJ'}$

The coefficients k'_{00} , k'_{11} and k'_{10} have been computed from numerical formulae given by Hamermesh & Schwinger (1947). Representative values are given in table 1. Hamermesh & Schwinger do not calculate k'_{01} . An expression for this coefficient was therefore obtained from equations (1) and (2). Numerical values are included in table 1.

TABLE 1. VALUES OF THE COEFFICIENTS $k'_{JJ'}$ ($T = 20.4^\circ \text{K}$)

inverse velocity of neutrons ($\mu\text{s/m}$)	k'_{00}	k'_{11}	k'_{10}	k'_{01}
633	4.197	4.253	1.153	0.118
745	4.581	4.616	1.161	0.018
856	4.869	4.891	1.192	0.003
1078	5.325	5.339	1.296	0.000
1300	5.709	5.721	1.431	0.000
1522	6.081	6.089	1.584	0.000

3. DESCRIPTION OF APPARATUS

(a) Introduction

The total cross-sections were determined by measuring the transmission ratio of a beam of neutrons passing through the gas. The scattering sample was placed in the beam and the neutrons that passed through undeflected were counted for a given time. The sample was then removed and neutrons in the uninterrupted beam were counted for a further period of time. A separate counter was used to monitor the beam.

If the number of counts with the sample in the beam is n_i , and the number, normalized to the same monitor count, with the sample out of the beam is n_0 , the transmission ratio R is given by

$$R = \frac{n_i}{n_0} = \exp(-N\sigma'_i x),$$

where N is the number of molecules per unit volume, σ'_i is the total cross-section per molecule and x is the length of the gas sample. The cross-section σ'_i is the sum of $\sigma_{\text{abs.}}$, the absorption cross-section, and $\sigma'_{\text{sc.}}$, the scattering cross-section. We thus have

$$\begin{aligned}\sigma'_i &= \sigma_{\text{abs.}} + \sigma'_{\text{sc.}} \\ &= \sigma_{\text{abs.}} + f_p \sigma'_{\text{para}} + (1 - f_p) \sigma'_{\text{ortho}}.\end{aligned}\tag{5}$$

The experiment was carried out with two mixtures of hydrogen, one normal hydrogen and the other almost pure *para*-hydrogen. These two measurements, together with the value of $\sigma_{\text{abs.}}$, suffice to determine σ'_{para} and σ'_{ortho} .

(b) Source of neutrons

The neutrons were obtained from a 37 in. cyclotron by means of the ${}^9\text{Be}(dn){}^{10}\text{B}$ reaction. They were slowed down in a disk of paraffin wax (A in figure 1) which was cooled by means of liquid oxygen. The neutrons emerging from the wax had a Maxwellian velocity distribution, and the range of velocities we required lay on the low energy tail of the distribution. Cooling the wax served to increase the number of neutrons in the required velocity range by a factor of 4. The apparatus used to select the neutrons according to their velocity, a time-of-flight instrument with ten channels, has been described elsewhere (Cassels 1951).

(c) Geometry of scattering apparatus

The arrangement of the apparatus is shown schematically in figure 1. The scattering sample was contained in the tank marked *B* which was mounted on a trolley so that the sample could be wheeled in and out of the beam. The collimating tubes *C* were aluminium cylinders lined with borax and surrounded by blocks of wax. Stray neutrons from the cyclotron were slowed down in the wax and then absorbed

in the borax. The neutron beam through the gas was defined by holes, 2 in. in diameter, cut in the cadmium sheets *D*.

The neutrons were detected by a BF_3 counter *E*, situated a few metres from the neutron source *A*. A second BF_3 counter *F* was used to monitor the beam.

FIGURE 1. Schematic arrangement of apparatus. *A*, disk of paraffin wax, source of slow neutrons; *B*, apparatus containing scattering gas; *C*, collimating tubes; *D*, cadmium sheets; *E*, neutron detector; *F*, neutron monitor; *G*, position of cadmium sheet during background measurements.

(d) *Scattering chamber assembly and associated apparatus*

The scattering chamber assembly is shown in figure 2. A central hollow copper cylinder, 3 in. in diameter and 32 in. in length, was sealed at the ends with aluminium caps, $\frac{1}{16}$ in. thick. It was surrounded by a concentric copper cylinder, 5 in. in diameter. The inner cylinder contained the scattering gas, and the space between the two cylinders was filled with liquid hydrogen. Surrounding the outer cylinder was a radiation shield, consisting of a copper tube 6 in. in diameter, to which were soldered liquid-nitrogen tanks. Aluminium plates, $\frac{1}{32}$ in. thick, were fixed across the ends of the shield. The entire assembly was mounted in a brass cylinder, 11 in. in diameter and 36 in. in length, which was pumped by an oil diffusion pump backed by a mechanical pump. Aluminium windows, $\frac{1}{16}$ in. thick, were made in the brass end-plates of the cylinder.

The inner assembly—gas chamber and liquid-hydrogen container—was supported by wires and springs attached to the shield. The shield assembly was then supported in a cradle that rested on the outer brass cylinder. To reduce the number of neutrons that reached the detector after being scattered in the gas, a lining of cadmium, $\frac{1}{32}$ in. thick, was inserted in the gas chamber.

With 10 l. of liquid hydrogen the inner assembly could be cooled from 80 to 20° K, and the scattering gas maintained at the latter temperature for about 25 h. A rough

measure of the liquid-hydrogen level was obtained by measuring the pressure difference between the top and bottom of the liquid hydrogen on an oil manometer. When the liquid-hydrogen container was full, this difference amounted to 1 cm of oil.

The scattering chamber assembly was mounted on a trolley which was moved by remote control. Also on the trolley were mounted a mercury manometer for measuring the pressure of the gas and a conventional circuit of glassware by means of which the gas was introduced into the scattering chamber and samples subsequently removed for analysis. The apparatus for preparing *para*-hydrogen was fixed to the side of the trolley.

FIGURE 2. Scattering chamber assembly. (Dimensions are in inches.)

(e) Preparation and analysis of scattering gas

The *ortho-para* composition of the gas was determined by thermal conductivity measurements. The apparatus used for the purpose is described in detail elsewhere (Stewart & Squires 1955). About 30 ml. (at n.t.p.) of the gas under investigation is introduced into a cylindrical glass cell surrounded by liquid nitrogen. A fixed current is passed through a tungsten wire stretched along the axis of the cylinder. The equilibrium temperature of the wire, found by measuring its resistance, depends on the thermal conductivity of the gas and this in turn depends on its *ortho-para* composition. The apparatus is calibrated by measuring the resistance of the wire for gases of known composition. The accuracy of the method was found to vary from 0.1 % for $f_p = 0.25$ to 0.25 % for $f_p = 1$. The way the cell was used to determine the composition of the scattering specimens differed in the normal and *para*-hydrogen experiments.

(i) Normal hydrogen specimen

Normal hydrogen was passed into the scattering chamber until the pressure reached the desired value. A sample of the gas was taken off at about hourly intervals and its composition was determined by means of the thermal conductivity cell.

The *para*-hydrogen cross-section contributes very little to the cross-section of normal hydrogen. Thus the measurement of the latter gives in effect the cross-section of *ortho*-hydrogen σ'_{ortho} . The error in the measurement of f_p was equivalent to an error of 0.2 % in σ'_{ortho} . As the total error in σ'_{ortho} was 0.7 %, the accuracy of f_p was adequate.

(ii) *Para-hydrogen specimen*

For almost pure *para*-hydrogen, the error in f_p from the thermal conductivity measurements was about $\frac{1}{4}$ %. This was quite inadequate for the *para* experiment because, the value of σ'_{ortho} being about 30 times that of σ'_{para} , an error of $\frac{1}{4}$ % in f_p gives an error of about 7 % in σ'_{para} . In the *para* experiment, the composition of the scattering specimen was therefore determined indirectly.

Para-hydrogen was prepared in the liquid phase by allowing 140 ml. of liquid hydrogen to stand over 45 g of charcoal to which 200 ml. of oxygen at n.t.p. had been previously added. The half-life of the conversion, about 15 min, was measured by analyzing samples of the vapour in the conductivity cell. (Evidence that the conversion proceeded exponentially is given in Squires & Stewart (1954), where a more detailed account of the conversion will be found.) The conversion was allowed to proceed for at least 20 half-lives. The fraction of *para* molecules in the liquid at the end of this time was thus very close to 0.9979, the equilibrium value at liquid-hydrogen temperature. The liquid was then vaporized and passed directly into the scattering chamber. The only part of the apparatus with which the vapour came into contact and which was not at liquid-hydrogen temperature was a clean glass siphon tube. Clean glass is known to produce negligible conversion even after several months. Thus the initial value of f_p for the gas in the scattering chamber was 0.9979, and as the temperature of the container was that of liquid hydrogen, f_p remained constant at this value.

From these considerations it is thought that, though the composition of the gas in the *para*-hydrogen experiment was not determined directly, it was known with adequate accuracy.

(f) *Background measurements and blank runs*

The effect of neutrons arriving at the detector by routes other than the direct one was obtained by placing a sheet of cadmium at the position marked *G* in figure 1. The number of neutrons counted by the detector in this condition was about 0.03 % of the number counted during normal runs.

The contribution of the aluminium end-plates to the measured cross-sections was found in two ways. The first was to repeat the transmission measurements with no gas in the specimen chamber. This method was used for all the normal hydrogen measurements and for about half of the *para*-hydrogen measurements. The second method was to place a dummy scattering chamber alongside the gas scattering chamber. The two chambers were similar in neutron transmission properties, but the dummy contained no scattering gas. Neutrons were counted with each chamber in the beam in turn. The difference between the two sets of counts was due solely to the scattering gas. Thus the effects of the end-plates were eliminated without the necessity of performing separate experiments.

4. ERRORS AND CORRECTIONS

The density of the hydrogen sample was obtained from its pressure and temperature with the aid of data published by the National Bureau of Standards (Woolley, Scott & Brickwedde 1948). The combined error in the density and the length of the sample was estimated to be 0.3 %. As previously mentioned, the uncertainty in the *ortho-para* composition of the hydrogen caused an error of about 0.2 % in the *ortho* experiment and a negligible error in the *para* experiment.

The energy of the incident neutrons was calculated from the distance and the time of flight. The uncertainties in these quantities were estimated to be equivalent to an uncertainty of 0.2 % in the total cross-section.

Statistical and systematic errors in neutron counting provided the major part of the total error. In the *ortho* experiment, the statistical error was 0.2 %. In the *para* experiment, owing to the low *para*-hydrogen cross-section and the consequent high transmission ratio, the statistical error amounted to 2.3 % of the scattering cross-section.

The variation of counting efficiency with counting rate was measured for both the detector and monitor by the method suggested by Melkonian (1949). As a result of these measurements, the experimental values of the *ortho* and *para* scattering cross-sections were increased by (0.2 ± 0.4) and (0.3 ± 0.6) % respectively.

The determination of the effects of the aluminium end-plates by the first of the two methods described in § 3 (*f*) is liable to a small systematic error, due to changes in the attenuation of neutrons by the atmosphere in the gas and blank measurements. The maximum error in the final cross-section due to these changes was estimated to be 0.2 and 0.5 % in the *ortho* and *para* scattering cross-sections respectively.

An estimate was made of the fraction of neutrons that reached the detector after being scattered, either elastically or inelastically, though a small angle. The effect of this type of scattering was found to be a decrease of about 0.1 % in the measured cross-sections.

The contribution of the errors discussed in this section to the total errors in σ'_{ortho} and σ'_{para} are summarized in table 2.

TABLE 2. SUMMARY OF ERRORS

source of error	magnitude of error (%) in	
	σ'_{ortho}	σ'_{para}
gas sample		
density }		
length }	0.3	0.4
<i>ortho-para</i> composition	0.2	nil
neutron energy	0.2	0.3
neutron counting		
statistics	0.2	2.3
variation of counter efficiency	0.4	0.6
blank measurements	0.2	0.5
total	0.7	2.5

5. RESULTS

(a) Normal hydrogen experiment

The normal hydrogen experiment was performed three times. The pressure of the scattering gas varied from 20 to 60 cm of Hg, with corresponding transmission ratios of about 0.5 to 0.2.

TABLE 3. EXPERIMENTAL VALUES OF $(a_t - a_s)^2$

inverse velocity of neutrons ($\mu\text{s/m}$)	$(a_t - a_s)^2$ (10^{-24} cm^2)	statistical error in $(a_t - a_s)^2$ (10^{-24} cm^2)
633	8.45	0.04
707	8.46	0.05
782	8.43	0.06
856	8.51	0.07
930	8.43	0.08
1004	8.46	0.05
1078	8.52	0.05
1152	8.53	0.06
1226	8.52	0.07
1300	8.37	0.08
1374	8.43	0.09
1448	8.39	0.11
1522	8.38	0.12
1596	8.64	0.13
1671	8.44	0.15
weighted mean	8.47	0.02

From equations (3), (4) and (5) we have

$$\sigma'_t = \sigma_{\text{abs.}} + (3a_t + a_s)^2 \{f_p k'_{00} + (1 - f_p) k'_{11}\} + (a_t - a_s)^2 \{f_p k'_{01} + (1 - f_p) (2k'_{11} + k'_{10})\}.$$

For $f_p \approx 0.25$, the term in $(a_t - a_s)^2$ amounts to about 96 % of σ'_t . $(a_t - a_s)^2$ has been calculated from σ'_t . The values for $\sigma_{\text{abs.}}$ and $(3a_t + a_s)^2$ used in the calculation were

$$v_n \sigma_{\text{abs.}} = (1.452 \pm 0.030) \times 10^{-19} \text{ cm}^3 \text{ s}^{-1} \text{ mol.}^{-1},$$

$$(3a_t + a_s)^2 = 0.58 \times 10^{-24} \text{ cm}^2.$$

No significant difference was found in the values of $(a_t - a_s)^2$ obtained at different gas pressures. The values of $(a_t - a_s)^2$, obtained at different gas pressures and the same neutron velocity, have been averaged. The average value for each neutron velocity is given in table 3.

The value of $(a_t - a_s)^2$ given by the weighted mean of all the normal hydrogen results is

$$(a_t - a_s)^2 = (8.470 \pm 0.056) \times 10^{-24} \text{ cm}^2.$$

The values of σ'_{ortho} calculated from the measured values of σ' are shown in figure 3.

(b) Para-hydrogen experiment

The *para*-hydrogen experiment was carried out on eleven occasions. The pressure of the scattering gas was about 60 cm of Hg on each occasion and the transmission ratio about 0.88. The dummy tank technique was used for the last six experiments. The results are given in figure 4, where σ'_t is plotted against the inverse velocity of the neutrons. With the value of $\sigma_{\text{abs.}}$ given above and the

FIGURE 3. Effective scattering cross-section of *ortho*-hydrogen against inverse velocity of neutrons. (Temperature of scattering gas 20.4°K.) ϕ , experimental point; —, theoretical curve based on

$$\begin{aligned}(a_t - a_s)^2 &= 8.47 \times 10^{-24} \text{ cm}^2, \\ (3a_t + a_s)^2 &= 0.578 \times 10^{-24} \text{ cm}^2,\end{aligned}$$

value $(a_t - a_s)^2 = 8.47 \times 10^{-24} \text{ cm}^2$, $(3a_t + a_s)^2$ may be calculated from σ'_t . The results are given in table 4. The weighted mean of these results is

$$(3a_t + a_s)^2 = (0.578 \pm 0.016) \times 10^{-24} \text{ cm}^2.$$

The absorption cross-section $\sigma_{\text{abs.}}$ is such a small part of the total cross-section of normal hydrogen that the value adopted has negligible effect on the value obtained for $(a_t - a_s)^2$. However, in the case of *para*-hydrogen, $\sigma_{\text{abs.}}$ is a much larger fraction of the total cross-section, and the value adopted has a considerable effect on the value obtained for $(3a_t + a_s)^2$. An increase of 1% in $\sigma_{\text{abs.}}$ results in a decrease of 0.5% in the value of $(3a_t + a_s)^2$.

The error in our value of $(3a_t + a_s)^2$ is 2.7%. This represents the combination of an error of 2.5% due to our measurements and an error of 1% due to the present uncertainty in the value of the absorption cross-section.

FIGURE 4. Effective total cross-section of 99.79 % *para*-hydrogen against inverse velocity of neutrons. (Temperature of scattering gas 20.4° K.) ϕ experimental point obtained with method of blank measurements (see § 3 (f)); $\blacklozenge$ experimental point obtained with the use of dummy scattering chamber; — theoretical curve based on

$$\begin{aligned}(a_t - a_s)^2 &= 8.47 \times 10^{-24} \text{ cm}^2, \\ (3a_t + a_s)^2 &= 0.578 \times 10^{-24} \text{ cm}^2, \\ v_n \sigma_{\text{abs.}} &= 1.452 \times 10^{-19} \text{ cm}^3 \text{ s}^{-1} \text{ mol.}^{-1}.\end{aligned}$$

TABLE 4. EXPERIMENTAL VALUES OF $(3a_t + a_s)^2$

inverse velocity of neutrons ($\mu\text{s/m}$)	$(3a_t + a_s)^2$ (10^{-24} cm^2)	statistical error in $(3a_t + a_s)^2$ (10^{-24} cm^2)
776	0.557	0.058
819	0.539	0.047
904	0.679	0.079
967	0.551	0.066
1032	0.670	0.104
1115	0.687	0.088
1197	0.611	0.027
1242	0.655	0.081
1263	0.560	0.112
1277	0.564	0.031
1322	0.646	0.098
1357	0.529	0.033
1437	0.561	0.039
weighted mean	0.578	0.013

6. DISCUSSION

We may compare our results with those of Sutton *et al.* (1947), who previously measured the cross-sections of *ortho*- and *para*-hydrogen and also with the results of other methods of measuring a_t and a_s .

The free proton cross-section σ_f is given in terms of a_t and a_s by

$$\sigma_f = 4\pi(\frac{3}{4}a_t^2 + \frac{1}{4}a_s^2),$$

which may be written in the form

$$\sigma_f = \frac{1}{4}\pi\{3(a_t - a_s)^2 + (3a_t + a_s)^2\}.$$

The term in $(a_t - a_s)^2$ amounts to about 98 % of σ_f . Thus measurement of σ_f , like that of the *ortho* or normal hydrogen cross-section, is effectively a measurement of $(a_t - a_s)^2$. In table 5 the values of the free proton cross-section, deduced from our results and from those of Sutton *et al.*, are given together with the results of Melkonian (1949), who has measured σ_f directly.

TABLE 5. FREE PROTON CROSS-SECTION (10^{-24} cm²)

<i>ortho-para</i> experiment	
Squires & Stewart	20.41 ± 0.14
Sutton <i>et al.</i>	19.7 ± 0.3
direct measurement	
Melkonian	20.36 ± 0.10

TABLE 6. COHERENT SCATTERING AMPLITUDE OF HYDROGEN (10^{-12} cm)

<i>ortho-para</i> experiment	
Squires & Stewart	$-(0.380 \pm 0.005)$
Sutton <i>et al.</i>	$-(0.390 \pm 0.012)$
crystal diffraction	
Shull <i>et al.</i>	$-(0.396 \pm 0.02)$
liquid mirror reflexion	
Burgy <i>et al.</i>	$-(0.378 \pm 0.002)$

The coherent scattering amplitude of hydrogen is defined by

$$f = 2(\frac{3}{4}a_t + \frac{1}{4}a_s).$$

Apart from the *para*-hydrogen experiment, two other experiments have been devised to measure f . They have the advantage of giving the sign as well as the magnitude of f . The first is that of Shull, Wollan, Morton & Davidson (1948), who measured the intensities of neutrons reflected from different planes in crystals of sodium hydride. From these intensities f may be calculated. Unfortunately the calculations are complicated by the thermal motions of the lattice points.

The second experiment, carried out by Burgy, Ringo & Hughes (1951), is to measure the angle of external reflexion of a beam of neutrons from a liquid hydrocarbon mirror. From these measurements and a knowledge of the coherent scattering amplitude of carbon, f may be calculated.

The results of these experiments are given in table 6. (Sutton's results have been recalculated so that they are based on the value of σ'_{abs} used in this paper.)

It can be seen from tables 5 and 6 that our results are in good agreement with the most accurate values of σ_f and f , which are those of Melkonian and Burgy *et al.*,

respectively. The values of a_t and a_s deduced from our results and from those of Melkonian and Burgy *et al.* are:

	a_t (10^{-12} cm)	a_s (10^{-12} cm)
Squires & Stewart	0.537 ± 0.004	$-(2.373 \pm 0.007)$
Melkonian and Burgy <i>et al.</i>	0.538 ± 0.002	$-(2.369 \pm 0.006)$

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