

### 1. Particle in 3D box

Consider a single free electron inside a cubic box of side  $L$ . Solving the time-independent Schrodinger Equation in cartesian co-ordinates using separation of variables, show that the allowed energy levels assuming periodic boundary conditions are

$$E = \frac{\hbar^2}{2m} \frac{4\pi^2}{L^2} (n_x^2 + n_y^2 + n_z^2)$$

where  $n_i \in I$ .

### 2. Connections between classical and quantum simple harmonic oscillators

- (a) Consider a quantum simple harmonic oscillator of energy in quantum state  $n$ . Show that the classical amplitude  $A$  corresponding to this state is  $A = \sqrt{(2n + 1) \frac{\hbar}{m\omega}}$ . As expected, the classical amplitude increases with the quantum number  $n$ .
- (b) By comparing the diagram given in class to result of (a) show for several  $n$  values that the quantum oscillator can be found in classically forbidden regions.
- (c) The probability of finding a quantum simple harmonic oscillator between  $x$  and  $x+dx$  is  $\rho(x)dx = |\psi(x)|^2 dx$ . Show that the probability of finding a classical oscillator between  $x$  and  $x+dx$  is

$$\rho(x) = \frac{dx}{\pi\sqrt{A^2 - x^2}}$$

- (d)  $\rho(x)$  is called the probability density. Qualitatively compare the shape of the classical and quantum probability densities, using the diagrams of the quantum simple harmonic oscillator wave functions given in class.