1. **Multiplicity**
   A system comprises N states which can have energy 0 or Δ. Show that the number of ways $g(U)$ of arranging the total system to have energy $U = r\Delta$ (where r is an integer) is given by

$$g(U) = \frac{N!}{r!(N-r)!}$$

   Now remove a small amount of energy $\epsilon = s\Delta$ from the system, where $s << r$. Show that

$$g(U-\epsilon) \approx g(U)\frac{r^s}{(N-r)^s}$$

   and hence show that the system has temperature T given by

$$\frac{1}{k_B T} = \frac{1}{\Delta}\ln\left(\frac{N-r}{r}\right)$$

   Hint: you will need to use Stirling's approximation:

$$\ln N! \approx N \ln N - N$$

2. **Boltzmann factor**
   Estimate $k_B T$ at room temperature, and convert this energy into electronvolts (eV). Using this result, answer the following:

   (a) Would you expect hydrogen atoms to be ionized at room temperature? (The binding energy of an electron in a hydrogen atom is 13.6 eV.)

   (b) Would you expect the rotational energy levels of diatomic molecules to be excited at room temperature? (It costs about 10^{-4} eV to promote such a system to an excited rotational energy level.)