

extra problems: phonons

1. Quantum and classical harmonic oscillators

A system of 200 quantum oscillators has 100 normal modes at each of two frequencies ω and 3ω where $\omega = 2 \times 10^{13}$ rad/sec. The system is in thermal equilibrium at temperature 300 K.

- (a) How many phonons exist in the system at this temperature ?
- (b) What is the vibrational energy of the system (excluding zero-pt energy) in eV ?
- (c) If these were classical oscillators, what would the total vibrational energy be in eV?
- (d) What happens to the number of phonons and the associated vibrational energy as the temperature decreases? increases ?
- (e) Does there exist any temperature range where the answer to (b) is greater than the answer to (c)? Prove your response.

2. Basis of two unlike force constants

Determine the dispersion relations $\omega(k)$ for lattice vibrations in a one-dimensional crystal ($2N$ atoms, lattice constant $2a$) with periodic boundary conditions. The unit cell consists of two identical atoms, but with different spring constants \mathbf{C} and $10\mathbf{C}$.