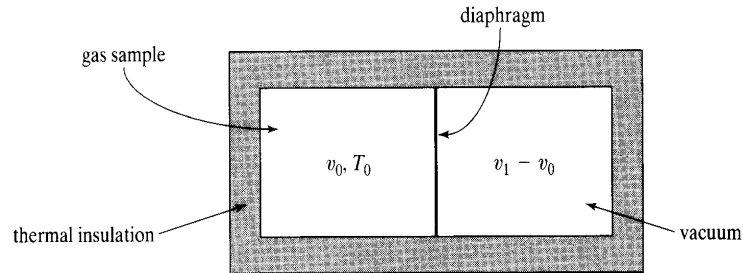


1. Gay-Lussac–Joule Experiment

In this experiment, an adiabatic chamber (thermally insulating walls which allow no heat transfer) with two compartments separated by a breakable diaphragm is constructed. We consider the state of the gas to be determined by (V, T) since P is fixed by the equation of state. Gas of volume V_1 , and temperature T_1 is introduced into one of the compartments while in the other there is a vacuum. When the diaphragm is broken, the gas undergoes a “free” expansion which does *NO* work. The gas has a new volume and potentially a new temperature which can be measured. However *NO* temperature change is observed!



(a) Show that this observation is consistent with the statement

$$\left(\frac{\partial U}{\partial V}\right)_T = 0$$

. Hint: You will need to use the multivariable calculus identities:

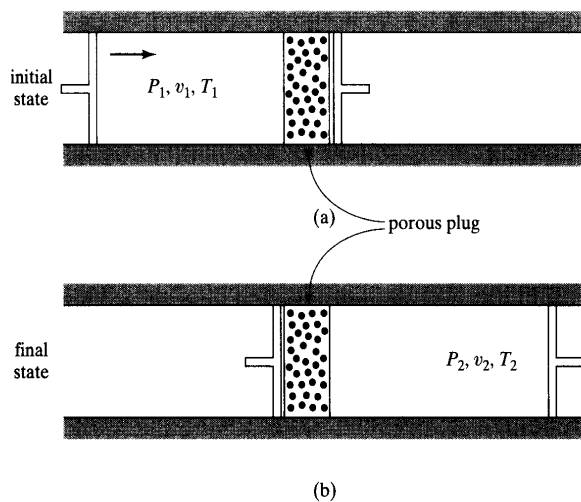
$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

$$1/\left(\frac{\partial y}{\partial x}\right)_z = \left(\frac{\partial x}{\partial y}\right)_z$$

(b) Show then that $dU = C_v dT$ in all cases.

2. Joule-Thompson Experiment

In this experiment, an adiabatic cylinder is constructed with a porous plug in the middle. A gas in state (P_1, V_1, T_1) is introduced on one side of the porous plug and held in place by a piston which is able to move and which can push the gas through the porous plug. Another piston is placed on the other side of the porous plug. Initially it is placed right up against the porous plug so that the volume of gas on the other side of the cylinder is 0. When the piston is pressed into the adiabatic metal cylinder, gas will be forced through the porous plug, and collect on the other side. When all of the gas has been pushed through, the gas on the other side of the plug is in state (P_2, V_2, T_2) . Experimentally, it is found that while $T_1 = T_2$ while $P_1 \neq P_2$.



- Show that this is a constant enthalpy process by calculating the total work done. Use the first law and note that this is an adiabatic process. When calculating the work done in this “throttling process”, assume that the pressures on either side of the plug are constant and equal to P_1 and P_2 , respectively.
- Show that the observation of no temperature change implies that $\left(\frac{\partial H}{\partial P}\right)_T = 0$. Again you need to use calculus identities listed above.
- Show then that $dH = C_p dT$ for any process.

3. Adiabatic Expansion

Prove that PV^γ is constant for an adiabatic process where $\gamma = C_p/C_v$.

4. Ideal Gas

(a) Show that

$$dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[\left(\frac{\partial U}{\partial V}\right)_T + P\right] dV$$

(b) Assuming that $\frac{dQ}{T} = dS$ is an exact differential, prove that

$$\left(\frac{\partial U}{\partial V}\right)_T = 0$$

for an ideal gas where the equation of state is $PV = nRT$.

5. An ideal diatomic gas, for which the *specific* heat capacities (i.e. per mole) are $c_v = 5R/2$ and $c_p = 7R/2$, occupies a volume of 2m^3 at a pressure of 4 atm and a temperature of 20°C . The gas is compressed to a final pressure of 8 atm. Compute the final volume, the final temperature, the work done by the gas, the heat released, and the change in internal energy for:

- (a) A reversible isothermal compression.
- (b) A reversible adiabatic compression.