Brock University Physics Department
St. Catharines, Ontario, Canada L2S 3A1

PHYS 1P93 Laboratory Manual

Physics Department

Contents

Laboratory rules and procedures

Physics Department lab instructors

Frank Benko, office B210a, ext.3417, fabenko@brocku.ca
Phil Boseglav, office B211, ext.4109, fbosegla@brocku.ca

This information is important to YOU, please read and remember it!

Laboratory schedule

To determine your lab schedule, click on the marks link in your course homepage; your lab dates are shown in place of the lab marks and correpond to the section number that you selected when you registered for the course. You cannot change these dates unless you have a conflict with another course and make the request in writing.

The schedule consists of five Experiments to be performed **every second week** on the same weekday. On any given day there are **four different Experiments** taking place, with up to three groups of **no more than two students in a group** performing the same experiment. You need to:

- **prepare for your scheduled experiment**. Out of schedule Experiments cannot be accomodated;
- **be on time**. The laboratory sessions **begin at 2:00 pm and end no later than 4:45 pm** and you will not be allowed entry once the experiments are under way.

Lab report format and submission

You are required to submit the Discussion component of your Lab Report to www.turnitin.com prior to the lab submission deadline. Instructions for registering and submitting your work are found in your course web page. Be sure to have a working account before you need to use it.

After you submit your Discussion to the Turnitin webpage, Turnitin will email to your Turnitin login email address (i.e. ab11cd@brocku.ca) a complete copy of the text that you submitted tagged with the submission date and a unique ID number. **Include this email** as part of your lab report.

Submit your report in the clearly marked wooden box across the hall from room MC B210a. **Reports are due by midnight one week after the experiment is performed**. For example, the report for an experiment performed on a Tuesday is due by midnight on the Tuesday following.

Compile your Lab Report as follows:

- submit the complete Lab Report in a **clear-front document folder**.

  Do not use three-hole Duo-tang folders, envelopes or submit a stapled set of pages;
- insert the first lab worksheet so that **your name is visible through the folder front cover**.

  Do not include a title page as the first experiment page is the title page;

- add the other lab worksheets in the proper sequence, followed by printouts and pages of calculations.
- **At the end of the Lab Report include a complete copy of the email sent by Turnitin**. This email contains your complete Discussion that will be graded.

  Do not submit instead printouts of the receipt from the Turnitin webpage or your wordprocessor; **these will not be graded and you will lose 40% of your lab mark!**
- **Note:** you should anticipate and be prepared for the likelyhood that Turnitin may not provide an immediate email response following your Discussion submission; this response may take several hours. Submit your work well ahead of the submission deadline.
- **Note:** Late Lab Reports will receive a zero grade, no exceptions.
- **Note:** Lab Reports not formatted as outlined will receive a 20 % grade deduction.
- **Note:** Marked Lab Reports will be returned to you during your next Lab session.

The lab manual

Your lab manual is available as a .PDF document in your course webpage. This allows you to print a copy of the experiment that you need for the current lab. It also allows the department to make quick edits to the manual to fix typographical errors, etc.

This lab manual contains five experiments. Each experiment consists of three components, and completing the lab means reaching all three of the milestones described below.

1. **Pre-lab review questions, to be completed before entry into the lab**, are intended to ensure that the student is familiar with the experiment to be performed. **A Lab Instructor will initial and date the review page** if the questions are answered correctly. The review questions contribute to your lab grade.

   - **You will be required to leave the lab if the review questions are not completed** as instructed. Missing your assigned lab date could result in a grade of zero for that Experiment.
   - Be sure to have a TA check and initial the completed review questions **before** you begin the lab. Lab reports missing the initials will be subject to a 20 % grade deduction.
   - In case of difficulties with any of the review questions, a student is expected to seek help from a lab instructor **well before** the day of the lab.

2. **A lab component is the actual performing of the experiment**. Marks are deducted for failing to complete all of the required procedures, follow written instructions, answer questions, provide derivations, the improper use of rounding and incorrect calculations. The lab report markers use a standard marking scheme to grade the lab reports.

   At the end of the lab session, if the lab procedures have been completed as required, **a Lab Instructor will also initial and date the front page** of your Experiment.

   - **An incomplete lab component will not be initialled**; you will need to finish the work on your time and have it signed before submission. A report missing this signature will be subject to a 20 % grade deduction.

3. **The final component is the compilation of the experimental data, its analysis, and a critical assessment of the results into a lab Discussion**. This component is worth 40 % of the lab mark.

The Discussion should consist of a series of paragraphs rather than an itemized list of one-line answers. You *do not need to* review the theory or reproduce formulas or tables of experimental data contained in the workbook as part of the discussion. You should:

- begin the discussion with a **tabulated summary of your data**, properly rounded according to the associated margin of error;
- thoughtfully answer and expand on the given guide questions, outline your observations, summarize the results of the experiment and support your conclusions with data or reasoned arguments;
- assess the validity of your results by comparing your values and their associated errors with values estimated from the theory or cited in your textbook or other literature.

Suggestions for improving the experimental procedure and a summary of the implications of the obtained results will make the discussion complete.

A guide to team collaboration

To ensure that the collaborative nature of the experimental team is expressed in a fair and mutually advantageous way for every member of the team:

- **Come prepared and ready to participate constructively** as part of your lab team!

- **Do not sit idle** and expect others to provide you with their data. The data gathering procedures should be undertaken by all the members of the team. While it may not be practical to have every student perform the same reading every time, each member of the team must become familiar with the equipment and perform some of the readings. The lab instructor will ask procedural questions during the lab and you will be expected to know what is going on in the experiment.

  All measurements are to be made by more than one student. This is a very effective way to verify a measurement; the use of an incorrect value in a lengthy calculation can waste a lot of your team's lab time and result in an incomplete lab. All labs finish by 4:45pm sharp.

- **Do your own calculations!**. There is sufficient time during the lab for this to be accomplished. As above, this is also a good strategy; comparing the results of several independent calculations can expose numerical errors and lead to the correct result or give you the confidence that your result is indeed correct. To access a calculator on your workstation, type *xcalc* in a terminal window.

- **Submit your own set of graphs**. Enter your own data, include your name and a description of the plotted data as part of the title. This approach will also expose any errors in the data entry or the computer analysis of the data. Needless to say, the Discussion section of the lab report is not to be a collaborative effort.

- **Warning: Do not copy** someone else's review questions, calculations or results. This is an insult to the other students, negates the benefits of having an experimental team and will not be tolerated. Any such situations will be treated as **plagiarism**. You should review in your student guide Brock University's definition and description of plagiarism and the possible academic penalties.

- **Warning: Do not allow others to copy** the content and results of your calculations or review questions; doing so makes you equally responsible under the definition of plagiarism. Do not feel pressured to allow another student to copy your work; inform the lab instructor.

Academic misconduct

The following information can be found in your Brock University undergraduate calendar:

”Plagiarism means presenting work done (in whole or in part) by someone else as if it were one’s own. Associate dishonest practices include faking or falsification of data, cheating or the uttering of false statements by a student in order to obtain unjustified concessions.

Plagiarism should be distinguished from co-operation and collaboration. Often, students may be permitted or expected to work on assignments collectively, and to present the results either collectively or separately. This is not a problem so long as it is clearly unbderstood whose work is being preented, for example, by way of formal acknowledgement or by footnoting.”

Academic misconduct may take many forms and is not limited to the following:

- Copying from another student or making information available to other students knowing that this is to be submitted as the borrower’s own work.
- Copying a laboratory report or allowing someone else to copy one’s report.
- Allowing someone else to do the laboratory work, copying calculations or derivations or another student’s data unless specifically allowed by the instructor.
- Using direct quotations or large sections of paraphrased material in a lab report without acknowledgement. (This includes content from web pages)

Specific to the Physics laboratory environment, you will be cited for plagiarism if:

- you cannot satisfactorily explain to the lab instructor how you arrived at some numerical answer entered in your laboratory workbook;
- you cannot satisfactorily describe to the lab instructor how you derived a particular equation in the lab procedure or as part of the review questions;
- your data is identical to that of some other student when the lab procedure stated that each student should obtain their own data.

The above points are based on the conclusion that if you cannot explain the content of your workbook, you must have copied these results from someone else.

In summary, you are allowed to share experimental data (unless otherwise instructed) and compare the results of calculations and derivations for correctness with other members of your group, but **the derivation of results must be your own work**.

Academic penalties

A first offence of **plagiarism in the lab** will result in the **expulsion** of all parties concerned from that lab session and the assignment of a **zero grade** for that particular lab. A record will be made of the event and placed in your student file.

A subsequent offence will initiate academic misconduct procedures as outlined in the Brock University undergraduate calendar.

Introduction to *Physica Online*

Overview

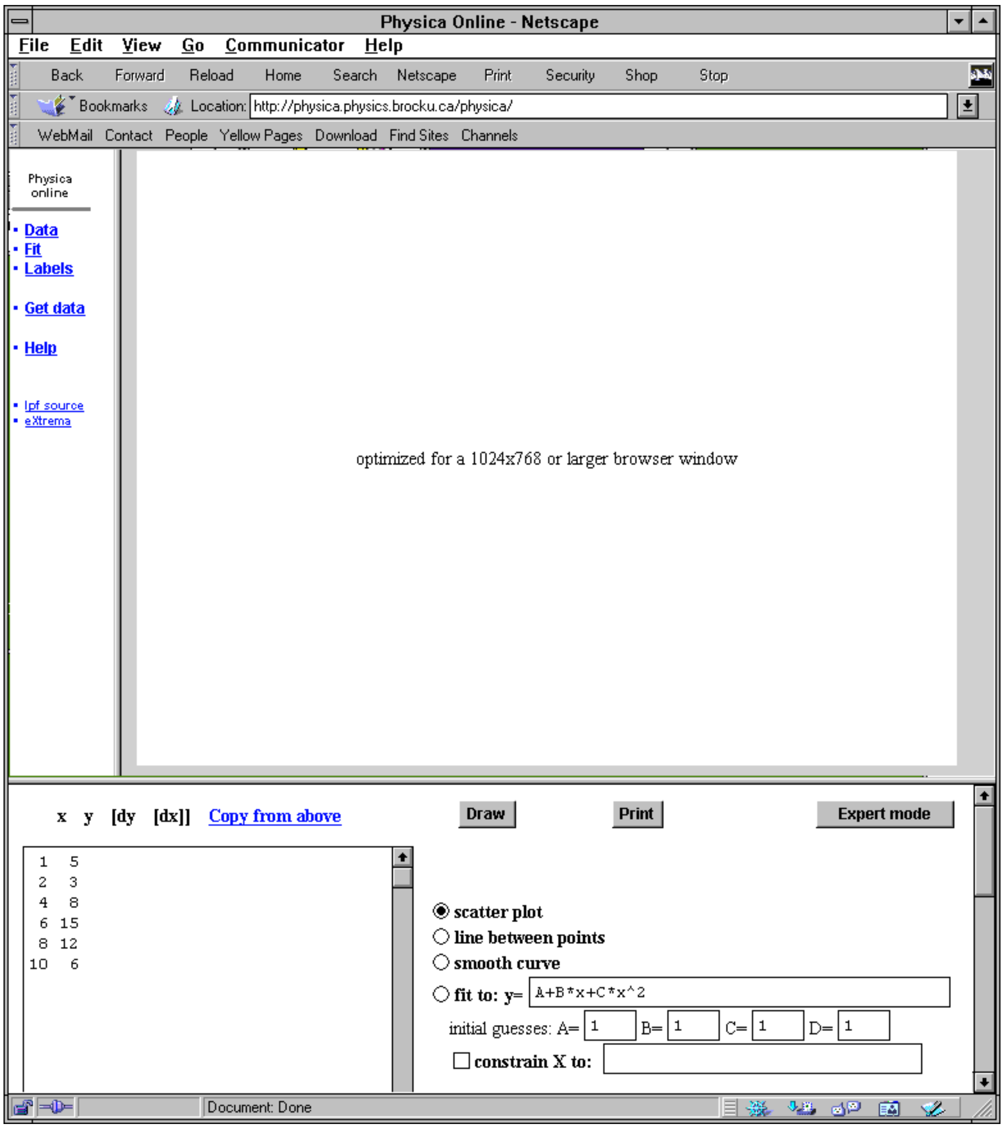

Figure 1: *Physica Online* opening screen

Physica Online is a web-based data acquisition and plotting tool developed at Brock University for the first-year undergraduate students taking introductory Physics courses with labs. It is accessible on the

Web at

http://www.physics.brocku.ca/physica/

When you first point your browser to this page, you should see approximately what is shown in Fig. 1. The "engine" behind the Web interface is the program `physica` written at the Tri-University Meson Facility (TRIUMF) in Vancouver. It is also available in a stand-alone menu-driven Windo$se version from http://www.extremasoftware.com/.

First and foremost, *Physica Online* **is a plotting tool**. It allows you to produce high-quality graphs of your data. You enter the data into the appropriate field of the Web interface, select the type of graph you want, and make some simple choices from the self-explanatory menus on the page. After that, a single button generates the graph. You can view it, print it, save it as a PostScript file for later inclusion into your lab report.

Physica Online **is a fitting and data analysis tool**. The `physica` engine has extensive and powerful fitting capabilities. Only a small subset is used in the "easy" default web mode, but it is sufficient for all of the experiments that Brock students encounter in their first-year labs. The full "expert" mode is also available for those needing more advanced capabilities of full `physica`; some learning of commands may be required.

Physica Online **is a data acquisition tool**. The web interface connects to a LabPro$^{\mathrm{TM}}$ by Vernier Software or a similar interface device, typically attached to a serial port of one of the thin clients[1] (or of some serial-port server). You can then copy-and-paste the returned data into the data field of *Physica Online*, ready to be plotted and/or analysed.

Below, we will follow the approximate sequence of a typical lab experiment. A demo mock-up of one such experiment (*RC* time constant determination) is available online, even from outside of the lab. It may be useful to open a browser, and point it to the demo *RC* lab while reading this manual.

Acquiring a data set

The data acquisition hardware consists of a variety of interchangeable sensors connected to a programmable interface device called a LabPro. This unit samples the sensors and transmits the data to a serial port of a thin client (or a personal computer).

To acquire a set of data from a sensor press **Get data** in the control panel of *Physica Online*. In the main plot frame to the right, a LabPro frame will open up, similar to the one seen in Fig. 2. In this frame you have to set several options by hand.

Begin by identifying the IP address of the thin client to which the LabPro hardware is attached. The thin client is identified as `ncdXX`, where `XX` should be set as indicated by the label on the terminal. Usually, it is the one you are sitting at, but sometimes you may need to use the LabPro attached to another thin client. Several groups of students can use the same hardware device to collect data, but not at the same time! In the example shown in Fig. 2, the **IP** is set to `ncd36`.

Next, specify one or more channels from which the data will be read. There are four available analog channels, **Ch1**–**Ch4**, used to attach probes of voltage, temperature and light intensity. The two digital channels, **Dig1** and **Dig2**, are used to connect probes such as photogate timers and ultrasonic rangefinders. More than one channel can be selected; in this case, more than two columns of data will be returned by the LabPro. In the example of Fig. 2, a single voltage probe is attached to **Ch1**.

Select the number of data points to be collected, the delay between successive data points and then initiate the data acquisition by pressing the **Go** button. Once the data collection begins, a progress message appears in this window indicating the time required to complete the data collection. Be patient

[1]Thin clients are desktop devices that provide a display, a keyboard, a mouse, *etc.*, but that do not have a disk or an operating system software. Instead, they connect to one of several possible servers, running whatever operating system that is required. The files and applications run on these servers, and the thin clients, or Xterminals, take care of the interactions with the user.

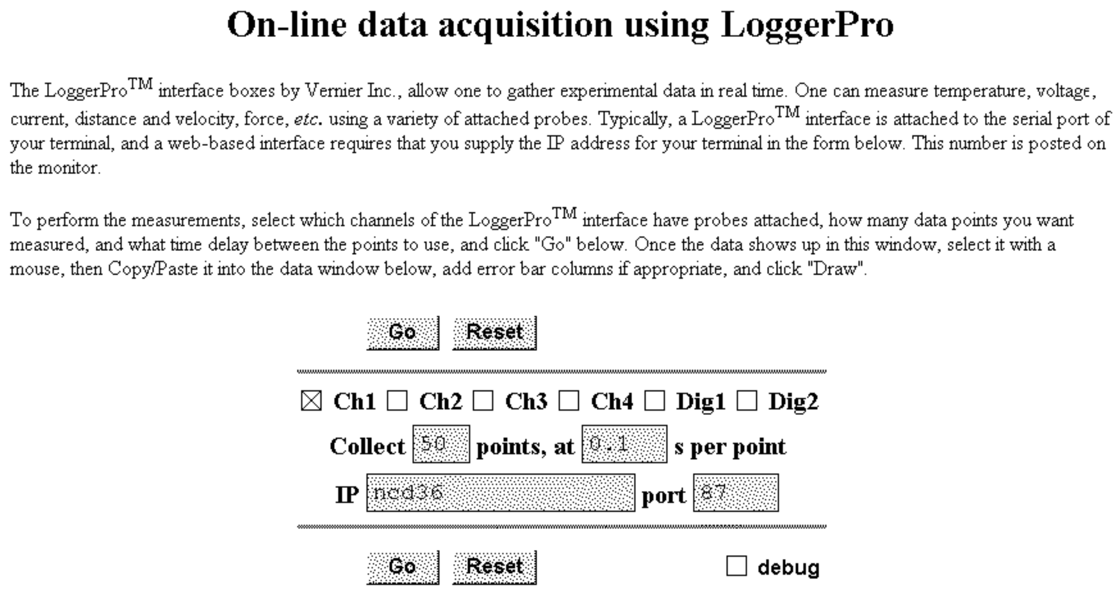

Figure 2: LabPro configuration frame

and let the LabPro process complete. If something is wrong and the browser is unable to communicate to the LabPro, it will time out after a few extra seconds of waiting. This may happen if the network is busy or if more than one browser is trying to obtain the data from the same LabPro; check that your **IP** field is set correctly.

Once the data acquisition is complete, you will have two columns of data in front of you. The next step is to select the data using your mouse, and copy-and-paste it to the data entry field below, as shown in Fig. 3. You can also do this by pressing the **Copy from above** button. You are now ready to plot and fit the data.

Graphing your data

The data entry field of *Physica Online* is usually filled through a copy-and-paste operation from the LabPro frame, as described in the previous section. You can also manually enter into this field any other data[2] that you wish to graph and analyse. You can press `Ctrl+A` to select all of the contents in the data window and `Ctrl+X` to delete those contents. On some machines, you need to use `Alt+A` and `Alt+X`.

The default settings are appropriate for generating a scatter plot of the data; all you need to do after entering or pasting in the data is to press **Draw**.

You have the option of associating error bars with each data point by entering one or two extra columns into the data field; the third column, if present, would be interpreted as Δy, and the fourth one, if present, as Δx. If the error is the same for all data points, you may use the the **dx:** and **dy:** fields below the data field. To omit the error bars, set these values to zero (this is the default).

There are several graphing options available. A **scatter plot** graphs a set of coordinate points using a chosen **data symbol** of a specific **size**. Check the **line between points** box to connect the data points with line segments or the **smooth curve** box to interpolate a smooth curve through the data points. After you made all your selections and pressed **Draw**, you may see the plot that looks similar to that shown in Fig. 4.

[2]Feel free to use *Physica Online* for preparing graphs for your other lab courses!

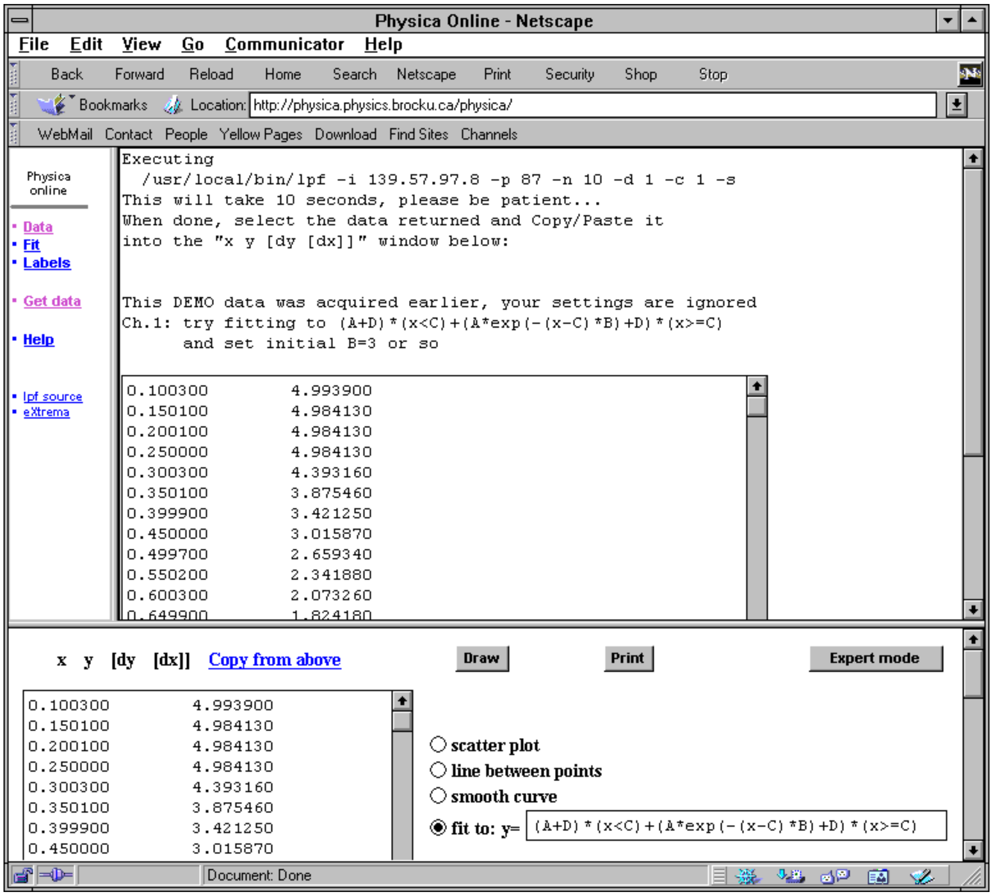

Figure 3: LabPro data has been copy-and-pasted into the data entry field

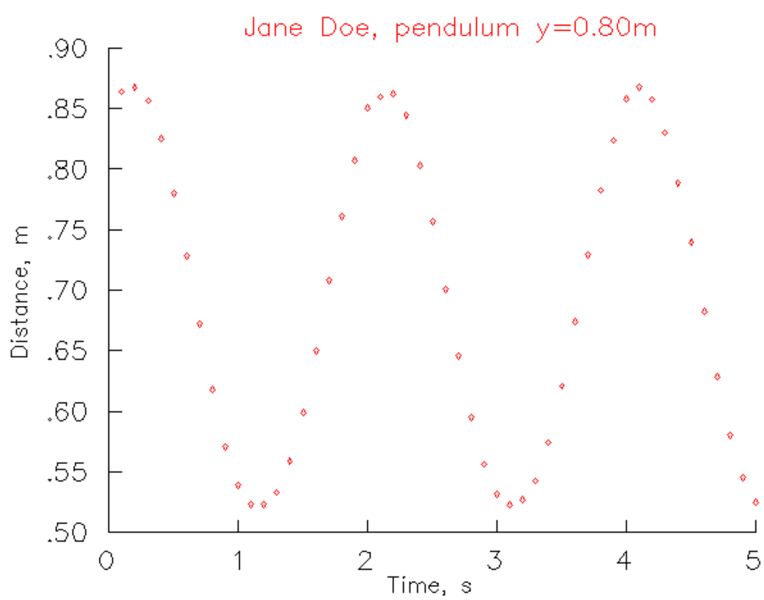

Figure 4: *Physica Online* data scatter plot

The **fit to: y=** option allows you to fit an equation of your choice to the data points. A second-order polynomial is pre-entered as the default, but in each experiment this will need to be changed to reflect the expected form of the $y(x)$ dependence. The simple web interface allows up to four fitting parameters **A, B, C, D**, which is enought for most of the first-year lab data. Much more elaborate fitting is possible from the "Expert Mode" of *Physica Online*. One essential point about fitting, especially when the fitting equation contains non-linear functions such as $\sin(x)$ or $exp(x)$, is to have a good set of approximate initial guesses for all fitting parameters. Examine the scatter plot of your data carefully, estimate the approximate values of all parameters your use in the fitting equation and enter your initial estimates in the fields provided. The default values of $A = B = C = D = 1$ are almost never going to work. If the fit fails to converge, *Physica Online* will return a text error message when you press **Draw**, you should then re-examine whether the fitting equation and the initial guesses for all parameters have been entered correctly.

You can fit more than one function to your data set, such as for example, a steady-state straight line followed by an exponential decay. These fits are explained further in the experiment in which they occur. You can also constrain the fit to two separate regions of your data set. In this case, you must copy-and-paste the fitting equation used in the **fit to: y=** box into the **constrain X to:** box or the error values will be incorrect.

Additional settings allow you to display the plot with the axes scaled in linear or logarithmic units, and the scale limits and increments can be manually set. A grid can be optionally included. The font and size of the text used to label the axes and in the title is also user selectable. Fig. 5 shows a set of values

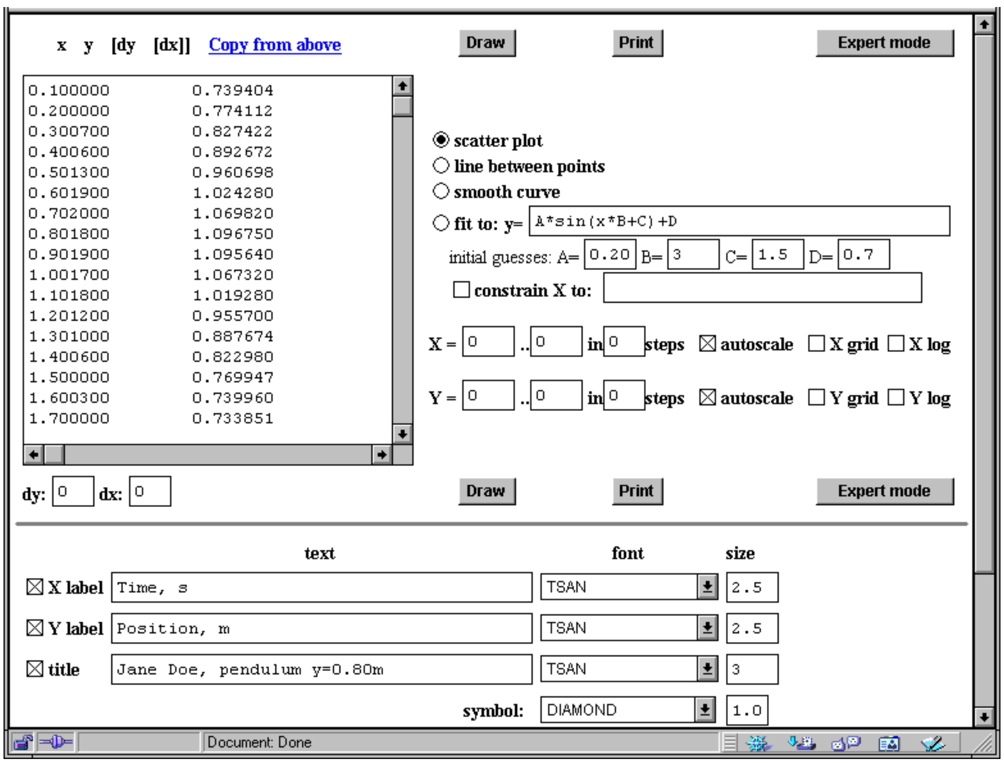

Figure 5: *Physica Online* fit and plot parameters

and settings that Ms. Jane Doe may have used for her data set. When she presses **Draw**, *Physica Online* returns the plot shown in Fig. 6.

An **Expert mode** button is available if other, more advanced features of `physica` are desired. Selecting this mode passes on all the settings from the "Easy Mode" and allows further changes to be made directly to the `physica` macro script. There is on-line help and tutorials, as well as hardcopy reference

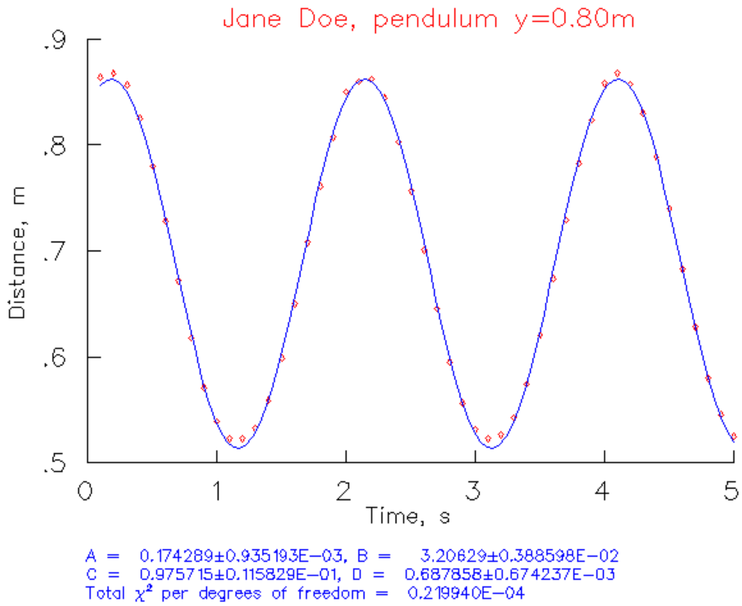

Figure 6: Experimental data and the fit, with settings from Fig. 5

manuals if you want to learn how to use these advanced features. For example, you may wish to plot two data sets on the same graph and add a legend. Feel free to change the macro; if you run into difficulties, press **Easy mode** and start again.

You can press **Print** to redirect the output to a PostScript printer. Only a few printer names are accepted as valid by the web script; your TA will tell you which one to use. If you leave the **Print to:** field blank, a PostScript file will be sent to your browser; if the browser knows how to display PostScript (though GhostView, or Adobe Acrobat, or a similar external application) it will do so. Otherwise, it should offer you an option to save it as a file; this is useful for later including your plot in a lab report or attaching it to an email.

Your browser's **Back** button will come in handy on occasion. If you find yourself hopelessly lost, you can also use **Reload** to bring you back to the starting point, although this will reset the graph settings such as title and axis labels to their default values.

FIRST NAME (PRINT)	LAST NAME (PRINT)	STUDENT NUMBER	TA INITIALS	GRADE

Experiment 1

Calorimetry and heat capacitance

Heat is a form of energy, and the *heat capacitance* of a body is a measure of the amount of this thermal energy that the body can hold. Heat capacitance depends on the material the body is made of, and measuring it can help to identify the material.

Heat naturally flows from hotter to colder bodies, and in general it is not easy to stop this flow of heat. There are several ways in which heat can be transported. One is direct contact of two bodies; the rate of direct heat *conduction* varies a great deal for different materials. Also, the rate of heat flow is roughly proportional to the temperature difference between the two bodies: the greater the difference, the faster the heat flow. For example, in very hot weather, as the temperature difference between the car's radiator and the air flowing past it is decreased, the effectiveness of cooling is diminished. When one of the bodies is a gas or a liquid that flow away from the other body after making a thermal contact, the heat is said to be transported by *convection*. When the differences in temperature are great, *radiative* heat transfer becomes important; all bodies above absolute zero emit thermal radiation, and the amount of this radiated energy grows very rapidly with temperature.

Even if the temperature is constant, heat energy may flow in and out of the system, if an internal rearrangement of atoms is taking place, such as a change of state from liquid to gas (evaporation) or from solid to liquid (melting). This so-called *latent heat* of evaporation or of melting again depends a great deal on the material in question, and can be used to identify the material.

Liquid nitrogen and safety

In this experiment, you will use liquid nitrogen as a handy source of a large temperature difference. Nitrogen gas becomes liquid at 77 K or −196°C, more than 200°C below room temperature. When liquid nitrogen comes into contact with room-temperature objects, a small amount of if evaporates very quickly and forms a thin layer of nitrogen gas. Since the heat conduction in gas is much slower than that in a solid or a liquid, this gas acts as a barrier to heat conduction, and it allows the remaining nitrogen to remain in liquid form. One could spill a small amount of liquid nitrogen directly onto skin, and live to tell the tale. But if a drop of liquid nitrogen is trapped in the folds of the skin, or if the skin comes into contact with a metal or glass container holding the liquid nitrogen, a serious injury will result. That is why you must:

Always wear gloves and goggles when handling liquid nitrogen!

Also, open-toed footwear will not be permitted in the laboratory!

Introduction to error analysis

The result of a measurement of a physical quantity must contain not only a numerical value expressed in the appropriate units; it must also indicate the reliability of the result. Every measurement is somewhat uncertain. Error analysis is a procedure which estimates quantitatively the uncertainty in a result. This quantitative estimate is called the *error* of the result. Please note that *error* in this sense is not the same as *mistake*. Also, it is not the difference between a value measured by you and the value given in a textbook. Error is a measure of the quality of the data that *your* experiment was able to produce. In this lab, *error* will be considered a number, in the same units as the result, which tells us the precision, or reliability, of that experimental result. **Note** that error value, represented by the Greek letter σ (sigma), *is always* rounded to one significant digit; the result *is always* rounded to the same decimal place as σ (see below).

Error of a single measurement

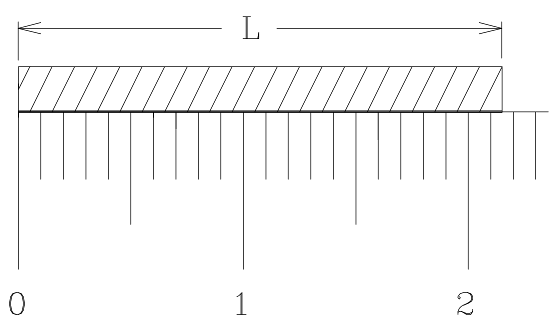

Figure 1.1: Measurement with a metre stick

Consider the measurement of the length L of a bar using a metre stick, as shown in Figure 1. One can see that L is slightly greater than 2.1 cm, but because the smallest unit on the metre stick is 1 mm, it is not possible to state the exact value. We can, however, safely say that L lies between 2.1 cm and 2.2 cm. The proper way to express this information is:

$$L \pm \sigma(L) = 2.15 \pm 0.05 \text{ cm}$$

This expression states that L must be between, $(2.15-0.05) = 2.10$ cm and, $(2.15+0.05) = 2.20$ cm, which is our observation. The quantity $\sigma(L) = \pm 0.05$ cm is referred to as the maximum error. This number gives the maximum range over which the correct value for a measurement might vary from that recorded, and represents the precision of the measuring instrument.

Propagation of errors

In many experiments the desired quantity, call it Z, is not measured directly, but is computed from one or more directly-measured quantities $A, B, C, \ldots$ with a mathematical formula. In this experiment, the directly-measured quantities are T, m and D, and the desired quantities ΔQ and C are calculated from $\Delta Q = 208 * D$ and $C = \Delta Q/(T * m)$. The following rules give a quick (but not exact) estimate of $\sigma(Z)$ if $\sigma(A)$, $\sigma(B)$ etc. are known Always use the *absolute value* of an error in a calculation .

1. If $Z = cA$, where c is a constant, then $\sigma(Z) = |c|\sigma(A)$. This is used *only* if A is a single term. For example, it can be used for $Z = 3y$, so that $\sigma(Z) = 3\sigma(y)$, but not for $Z = 3xy$.

$$\text{If} \quad \Delta Q = 208 * D \quad \text{then} \quad \sigma(\Delta Q) = 208 * \sigma(D)$$

2. If $Z = A + B + C + \cdots$, then $\sigma(Z) = \sigma(A) + \sigma(B) + \sigma(C) + \cdots$. For example, if

$$\begin{aligned} y &= y_0 + \frac{1}{2}\,y_1 \\ \text{then} \quad \sigma(y) &= \sigma(y_0) + \sigma\left(\frac{1}{2}y_1\right) \text{ (See 2. above.)} \\ \sigma(y) &= \sigma(y_0) + \frac{1}{2}\,\sigma(y_1) \quad \text{(See 1. above.)} \end{aligned}$$

3. If $Z = \dfrac{A^p B^q}{C^r D^s}$ with p, q, r, and s constant (but not necessarily integers), then

$$\frac{\sigma(Z)}{|Z|} = |p|\frac{\sigma(A)}{|A|} + |q|\frac{\sigma(B)}{|B|} + |r|\frac{\sigma(C)}{|C|} + |s|\frac{\sigma(D)}{|D|}.$$

In general, to derive an error equation for any relation, it is helpful to rewrite that relation as a series of multiplications, i.e.

$$g = \frac{4\pi^2 L}{T^2} \longrightarrow g = 4\pi^2 L T^{-2}.$$

The error equation for this relation is then

$$\frac{\sigma(g)}{g} = \frac{\sigma(4)}{|4|} + 2\left(\frac{\sigma(\pi)}{|\pi|}\right) + \frac{\sigma(L)}{|L|} + |-2|\left(\frac{\sigma(T)}{|T|}\right).$$

The quantities 4 and π are constants and have no error (strictly speaking, $\sigma(4) = \sigma(\pi) = 0$), therefore these terms do not contribute to the overall error. The error equation then becomes

$$\frac{\sigma(g)}{|g|} = \frac{\sigma(L)}{|L|} + 2\left(\frac{\sigma(T)}{|T|}\right).$$

The right hand side of equation (3), called the "relative error" of g, results in a fraction that describes how large $\sigma(g)$ is with respect to g. The desired quantity, $\sigma(g)$, can be obtained by multiplying both sides of the equation by g;

$$\sigma(g) = |g|\left[\frac{\sigma(L)}{|L|} + 2\left(\frac{\sigma(T)}{|T|}\right)\right].$$

Rounding

The value of $\sigma(x)$ is rounded to one significant digit whether it represents a maximum error estimate, calculated error, or standard deviation of a sample. The result corresponding to this error must be rounded off and expressed to the same decimal place as the error. For example, $\langle x \rangle = 25.344$ mm and $\sigma(x) = 0.0427$ mm. Rounded to one digit, $\sigma(x) = 0.04$ mm. Rounded to the same decimal place, $\langle x \rangle = 25.34$ mm. The final result is expresses as $\langle x \rangle \pm \sigma(x) = (25.34 \pm 0.04)$ mm.
Do not use a rounded off value in further calculations. Use the original unrounded value. Use of a truncated value will decrease the quality of your result.

Powers of 10

It is helpful to express both the result and its error to the same power of 10. This allows the reader to immediately judge how large the error is relative to the result:

1. $2.68 \times 10^{-2} \pm 5 \times 10^{-4}$ should be written as 0.0268 ± 0.0005 or, preferably, $(2.68 \pm 0.05) \times 10^{-2}$. Note the parentheses, indicating that both the result and the error are to be multiplied by 10^{-2}, not just the error.

2. $1.634 \pm 3 \times 10^{-3}$ m should be written as 1.634 ± 0.003 m

Format of calculations

Record all calculations, in the appropriate space if provided or on a separate sheet of paper. A calculation is performed in three lines. The first line displays the formula used. In the second line, the variables in the formula are replaced with the actual values used in the calculation. The third line shows the final answer properly rounded and if any, the units associated with the result.

The digital weight scale

A computer-connected precise digital weight scale will allow you to monitor the weight of a Styrofoam cup containing liquid nitrogen. As the heat is slowly transferred from the room-temperature environment, or as we add a known amount of room-temperature material into the cup, the added heat will turn some of this liquid nitrogen into gas. As a result, the mass reported by the scale will decrease as a function of time. As you analyze the nature of this time dependence, you should be able to extract several relevant physical quantities with considerable precision.

Simply start the Physicalab data acquisition program, then select `adam` as the input channel and click **Get data** to acquire and graph a data set.

Review questions

- Cooks test the hotness of the frying pan by spilling small droplets of water onto the pan's surface. On a warm frying pan, the drops quickly evaporate. When the pan becomes very hot, the drops survive much longer [*sic!*], and tend to "dance" around the surface of the frying pan for a while, before evaporating. Explain.

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- In many physical situations, the rate of change of some quantity is proportional to that quantity itself

  $$\frac{\Delta A}{\Delta t} \propto A \qquad \text{or} \qquad \frac{dA}{dt} \propto A \quad .$$

  For example, the rate of heat transfer and, hence, the rate of change of temperature is proportional to the temperature difference between the two bodies. If one of the bodies is much larger than the other and can be assumed to maintain a constant temperature (the so-called "temperature bath") then the rate of change of temperature of the smaller body is proportional to its temperature. This behaviour is known as Newton's Law of Cooling.

  What kind of a function describes the resulting time dependence, $A(t)$? (Hint: What is Newton's Law of Cooling?)

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`My Lab dates:  Exp.1:.......  Exp.3:.......  Exp.4:.......  Exp.5:.......  Exp.6.......`

<u>*CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!*</u>

Procedure and analysis

- Turn on the digital weight scale, then gently place several small objects: a coin, an eraser, an empty Styrofoam cup, *etc.* onto the scale. Note the reported mass of the objects, how quickly the scale reading settles to a constant value, the precision of the scale. Note what effect vibrations due to your footsteps or leaning of the table have on the readings, and plan the rest of your experiment accordingly.

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Wear gloves and protective eye-wear!

- Fill the cylindrical Styrofoam cup to about two thirds with liquid nitrogen, then place it on the scale. Shift focus to the Physicalab software. Set the input channel to **Adam**, select **scatter plot** then choose to acquire a data point every 5 seconds for at least 20 minutes. For a quick look at the data you can press the **Draw** button.

- Try a simple linear fit, `A*x+B`, and note the resulting χ^2 misfit value. Over short time intervals, a linear function might be appropriate, but as the level of liquid nitrogen in the cup changes, so does the slope of the curve $m(x)$, where x represents time. Make a copy of the graph by pressing the **Print** button. Each student should include a copy of every graph in their lab report.

  Now try an exponential fit, `A*exp(-x/B)` and note again the χ^2 misfit. Your plot should look similar to the one shown in Fig. 1.2.

  The parameter `B` in the exponential fit has the dimension of

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  and its physical meaning could be described as

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  `B` = ... ± . . . .

  If the liquid-nitrogen level falls below approximately one-third-full, the exponential function may become insufficient to describe the time dependence you observe, and an additional constant offset

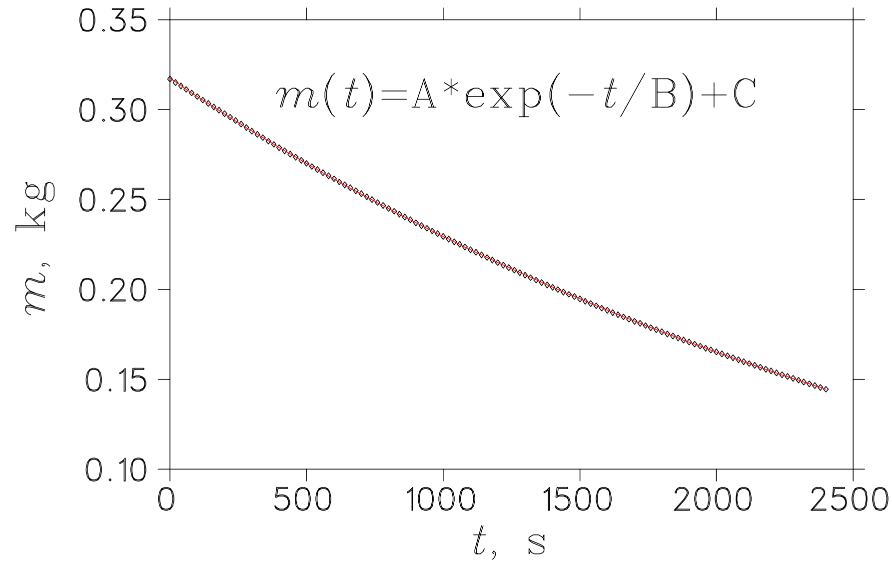

Figure 1.2: Mass of a cup of liquid nitrogen decreases with time due to boil-off

parameter may need to be added to the exponential function. Try fitting to `A*exp(-x/B)+C` and make a note of the reported χ^2 values.

The parameter `C` has the dimension of

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and its physical meaning could be described as

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`C` = ... ± . . . .

Use paper towels to wipe down the frost and accumulated condensation from the Styrofoam cup and the digital scale plate.

- Determine the weight of the provided brass block with the highest possible precision.

m_{brass} = ... ± . . . .

Use the room thermostat thermometer to determine the "room temperature" which is the initial temperature of the brass block. Avoid prolonged handling of the block to prevent it from absorbing your body heat.

T_{room} = ... ± . . . .

Wear gloves and protective eye-wear!

- Refill the Styrofoam cup to two-thirds-full and place it and a small Styrofoam piece with the brass block on top of it onto the weight scale. Start your data acquisition run; aim to get a point every 5 seconds over approximately 15–20 mins.

  Five minutes should be enough to establish the initial slope of the mass-as-a-function-of-time curve.

- Quickly transfer the brass block into the Styrofoam cup. Be careful to prevent splashing and avoid touching the surface of the liquid nitrogen with anything but the brass block itself. You can expect a significant cloud of cold gas to be generated when the brass is rapidly cooled by the liquid nitrogen to 77 K.

  Try to perform this transfer in between the data points to avoid picking up erratic readings; this is not essential, however.

  Continue the data acquisition until the rapid boil-off stops (2–3 mins) and for five minutes afterwards, to establish the trailing slope of the curve.

  At the end of the run the brass block is at 77 K and is **extremely dangerous**. Carefully transfer the brass block into the sink and run tap water over it until all traces of ice are gone. Wipe dry and return it to your station.

- Transfer the data set to *Physica Online* and establish through a visual inspection of the graph the two time points T_1 and T_2 that correspond to the beginning and the end of the rapid boil-off interval.

- Fit your data to `(A*exp(-x/B)+C)*(x<`T_1`)+((A-D)*exp(-x/B)+C)*(x>`T_2`)` using the values of T_1 and T_2 determined above. To get a valid χ^2 value, the constraint equation `(x<`T_1`)*(x>`T_2`)` must be entered in the constraint box. Examine the quality of the fit, and note the χ^2 misfit value.

- Repeat several times, changing the values of T_1 and T_2 slightly to move away from the edges of the region of the rapid boil-off. Ideally, the value of `D` reported by the fit should not depend on the precise choice of T_1 and T_2.

T_1, s	T_2, s	`D` $\pm\Delta$`D`	χ^2
	Best D value: (least χ^2)		

$D = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

`D` represents the "extra" liquid nitrogen boiled off to cool down the brass block from T_{room} to 77 K. It takes 208 kJ to boil off 1 kg of liquid nitrogen. Your task is to calculate the *average* specific heat of brass, c_{brass} *i.e.* the amount of heat that is required to raise the temperature of 1 kg of brass by 1 K.

- Convert `D` into the total amount of heat ΔQ

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$\Delta Q =$ . . . . $\pm$ . . . .

- Calculate the heat capacity of brass c_{brass} and the associated error using $\Delta Q = c_{\text{brass}} m_{\text{brass}} \Delta T$. This result is only an average value because the heat capacitance is not constant in brass between 77 K and the room temperature.

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$c_{\text{brass}} =$ . . . . $\pm$ . . . .

Compare your result to the accepted value, $c_{\text{brass}} = 0.093 \pm 0.002 \, \frac{\text{cal}}{\text{g °C}}$ at 20°C.

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IMPORTANT: BEFORE LEAVING THE LAB, HAVE A T.A. INITIAL YOUR WORKBOOK!

Discussion

To complete this lab, submit a typed or neatly written Discussion of the results of the experiment (double-spaced, 200 to 400 words). Attach your computer printouts and worksheets to this Discussion. Discuss the following issues as part of your Discussion; as always this is not a complete list:

- What is the likely dominant mechanism of heat transfer from the room-temperature environment to the cup of liquid nitrogen?
- Is Newton's Law of Cooling obeyed? Explain your answer.
- What effect does condensation (frost on the walls of the Styrofoam cup) have on the results?

A Final Note: Have you printed and included the **complete email that Turnitin sent you** containing the full content of your Discussion? If not, you will lose 40% of your grade. Printouts of the receipt from the Turnitin webpage or your wordprocessor will not be considered for marking.

FIRST NAME (PRINT)	LAST NAME (PRINT)	STUDENT NUMBER	TA INITIALS	GRADE

Experiment 2

Check your schedule!

This is a reminder that there is no Experiment 2 and that *you need to check your lab schedule* by following the marks link in your course homepage to determine the experiment rotation that you are to follow. The lab dates are shown in place of lab grades until an experiment is done and the mark is entered.

```
My Lab dates:  Exp.3:.........  Exp.4:.........  Exp.5:.........  Exp.6.........
```

Note: The Lab Instructor will verify that you are attending on the correct date and have prepared for the scheduled Experiment; if the lab date or Experiment number do not match your schedule, or the review questions are not completed, you will be required to leave the lab and you will miss the opportunity to perform the experiment. This could result in a grade of Zero for the missed Experiment.

To summarize:

- There are five Experiments to be performed during this course, Experiment 1, 3, 4, 5, 6.
- Everyone does the first experiment on the first scheduled lab session.
- The next four experiments are scheduled concurrently on any given lab date.
- To distribute the students evenly among the scheduled experiments, each student is assigned to one of four groups, by the Physics Department. The schedule is entered as part of your lab marks.

Lab make-up dates: You may perform **one** missed lab on April 2, 3, 4, 5, or 6.

Notes : ....

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Experiment 3

The Stirling engine

It is said that the plight of poor workmen, often injured by the explosions of the early steam engines were the inspiration for Rev. Robert Stirling's engineering efforts that led to the development of what we now call "the Stirling engine" in 1816. However, Rev. Stirling's life-long interest in engineering, his father's profession, suggests that the technical challenges fascinated him as well. He is the author of several patents, from the "Heat Economizer" (regenerator) to various optical devices, his "air engine" being the most famous.

Stirling engines are typically *external* combustion engines, as opposed to the open-cycle internal combustion engines prevalent today. The Stirling cycle involves moving a working fluid, typically a gas, between two reservoirs that are maintained at different temperatures, mediating the heat transfer from the hot to the cold side and extracting mechanical work in the process. In this way, the Stirling engines are very similar to the idealized heat engines textbooks on Thermodynamics. The designs of Stirling engines vary considerably, but they all share the inherent safety of a much lower operating pressure of the working gas, and fairly high heat-to-work conversion efficiency, especially compared to a standard gasoline engine. Their dynamic properties — the ability to rev up quickly, for example — are quite poor, so they are not well suited for motor vehicle use, but there are marine engines, hand-held toys, and deep-space probe electrical generators that all use Stirling engines.

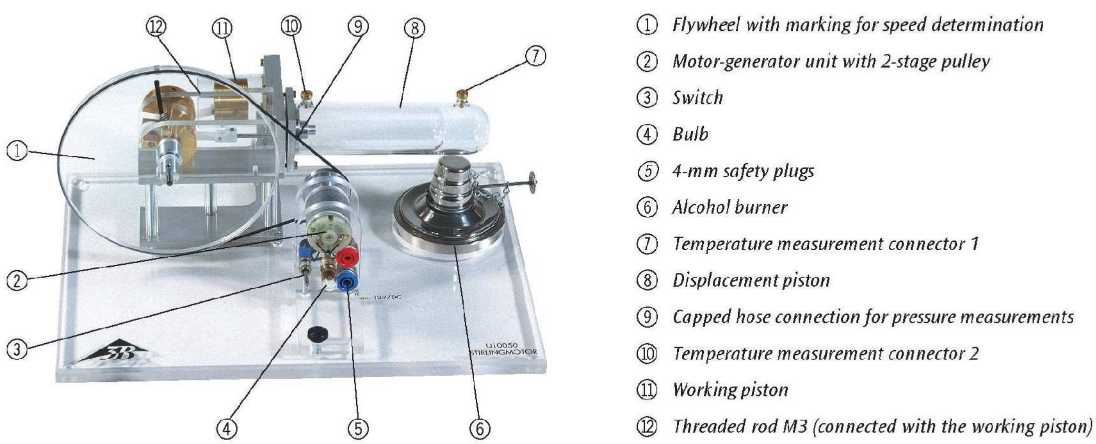

Figure 3.1: The U10050 Stirling Motor. Reproduced from the 3B Scientific manual.

The engine used in this lab is a displacement-type engine that uses two separate pistons, connected to a common flywheel (see Fig. 3.1). One piston is the working piston; it fits snugly into the cylinder and maintains the seal on the working gas. The other, a displacement piston, is moving freely inside its cylinder, shifting the working gas through the gap between the piston and the cylinder from the hot to the cold end of the cylinder, and back again. The full Stirling cycle can be approximately divided into four parts:

1. the gas is shifted to the hot end by the displacement piston; it expands as it heats up, and pushes the working piston which in turn rotates the flywheel;
2. the displacement piston moves, shifting the gas toward the cold end; heat is lost from the gas;
3. cooled gas is re-compressed by the inertia of the flywheel pushing the working piston.
4. with the gas compressed by the working piston, the displacement piston shifts it to the hot end, where the heat is absorbed into the gas.

The four parts of the cycle are illustrated schematically in the left-hand frame of Fig. 3.2. Since it takes

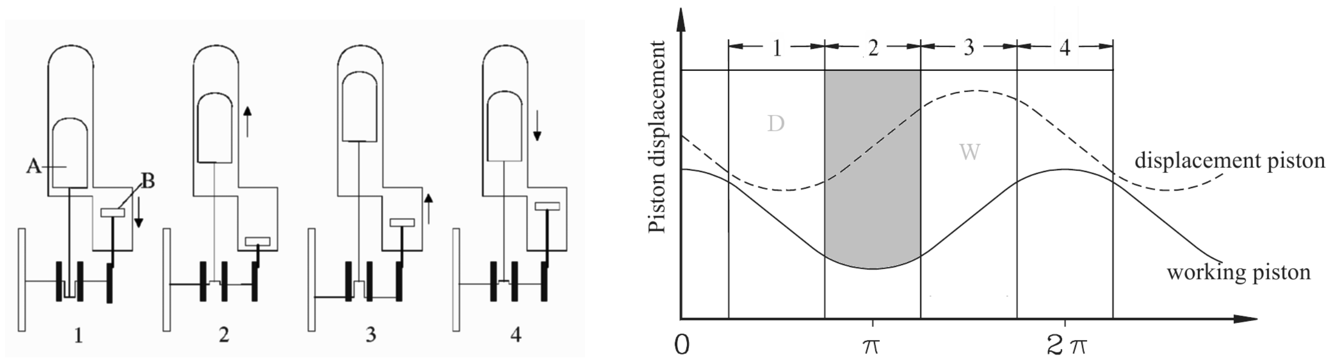

Figure 3.2: The Stirling cycle. **Left:** Four snapshots 1–4 show the relative positions of the displacement (**A**) and working (**B**) pistons that correspond approximately to the four parts of an idealized Stirling cycle. **Right:** A more realistic plot of the positions of the two pistons during the continuous rotation of the flywheel. The grey region in part 2 shows the part of the cycle where the working piston is near its lowest point and the gas volume is approximately at its maximum. The area above the dashed line represents the fraction of the total volume that is above the displacement piston, in thermal contact with the hot end of the cylinder. Reproduced from the 3B Scientific manual.

less work to compress the gas when it is cold (against lower gas pressure) than the work done by the gas on the piston when it is hot (gas exerting higher pressure over the same piston travel distance), there is a net conversion of heat into mechanical work.

Dividing the cycle into four discreet parts is an idealization. A more realistic plot is shown in the right-hand frame of Fig. 3.2, where the positions of both pistons vary continuosly throughout the cycle. Both pistons are attached to the same flywheel, but at 90° to each other. As a consequence, the solid and the dashed line are a quarter of a cycle out of phase with each other.

Fig. 3.3 illustrates the thermodynamics of the Stirling cycle. As we follow along the curve made by the measured pairs of (p_i, V_i), the area on the pV-diagram inside the cyclical trajectory of the working cycle of the engine represents the amount of mechanical work performed by it:

$$\mathcal{W} = \sum_i F_i \Delta x_i = \sum_i (p_i A) \Delta x_i = \sum_i p_i (A \Delta x_i) = \sum_i p_i \Delta V_i$$

The work is being done by the gas on the flywheel along the upper curve, and by the flywheel on the gas along the lower curve; the difference is the net work done by the engine. In our example, one can estimate

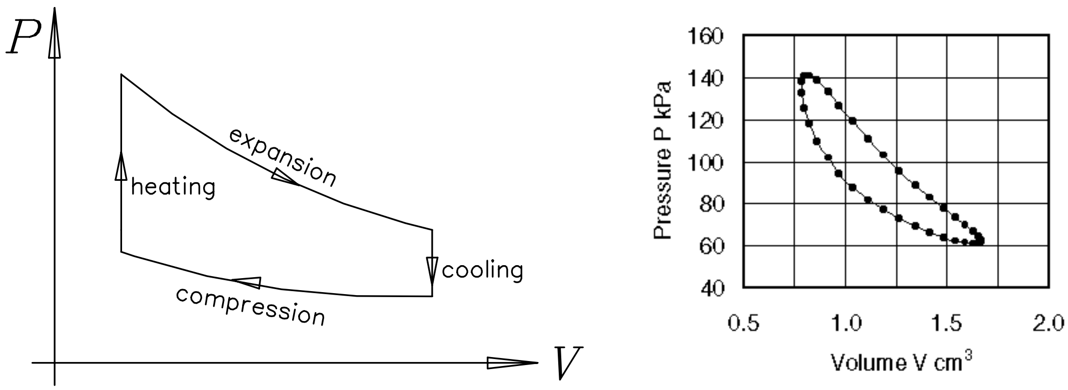

Figure 3.3: The pV-diagrams of the Stirling cycle. The idealized cycle in the left frame is made up of four distinct parts. In a realistic Stirling engine the parts of the cycle are not completely separate, and the pV-diagram looks more like the one in the right frame.

graphically the area inside the curve of Fig. 3.3 (right frame) to be about three–four large squares, 20 kPa by $0.25\text{cm}^3 = 0.25 \times 10^{-6}\text{m}^3$, and therefore, about 15–20 mJ per cycle.

Stirling engines, in general, have excellent coefficients of performance (CoP), reaching as high as twice the thermal efficiency of internal-combustion gasoline engines. When used to drive electric generators, as in this experiment, the thermal efficiency, though high, is just one of many of the CoP's that need to be multiplied together to get the overall CoP: that of the mechanical efficiency of the flywheel-piston system, that of the conversion efficiency of mechanical into electrical energy, *etc.* The overall efficiency of the process of conversion of the internal chemical energy of the fuel into the usable electrical energy that can turn on a lighbulb, is only of the order of 10%. Increasing the difference in the operating temperatures of the hot and cold ends, changing the working gas, or the compression ratio, can dramatically affect that number, but the cost and complexity of the engine may also increase.

Stirling engines, long a domain of tinkerers and enthusiasts, are making a comeback. Six Swedish submarines are equipped with Stirling engine propulsion units, enabling them to conduct long-range dives fueled by a chemical source of energy that does not involve a conventional burning of liquid fuel, without the use of nuclear power. Long-range space probes make use of a Stirling-cycle electric power generators fueled by the heat of radioisotope heat sources. Combined heat-and-power generators based on a Stirling engine are commercially available in the 100 kW power range, and their miniature cousins may one day replace laptop and cell phone batteries.

The many advantages of the Stirling engines are well described in the excellent Wikipedia article (recommended reading), one of which is their flexibility: their ability to act as heat pumps as well as engines. After all, if one uses an external motor to drive the Stirling engine through its thermal cycle, the net effect is the reverse conversion from mechanical energy into thermal one, heating up one end and cooling down the other end of the cylinder: following the pV-curve of Fig. 3.3 in the counterclockwise direction means a change of sign of the difference between the upper and lower halves of the cycle. Even more interesting is the ability of the Stirling engine to reverse the direction of this heat flow, if the direction of rotation of the engine is reversed. Thus the same Stirling-cycle thermal pump can act as a heat source in winter and a cooling device in the summer!

Review questions

For a hypothetical Stirling engine rotating at 512 rpm, the measured area between the upper and lower curves on the pV-diagram was found to be 47.7 kPa·cm^3 (see Fig. 3.3). Calculate the mechanical power, in Watts.

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The engine is used to drive an electrical generator that operates a lightbulb at 9.6 V with a current of 410 mA going through it. Calculate the electrical power (the product of voltage V and current I) used by this circuit.

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Calculate the CoP of this engine, in %.

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CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Procedure and analysis

In this experiment, the Stirling engine is used as a heat pump, the opposite of a heat engine. A DC voltage is applied to the electric motor that rotates the flywheel and moves the pistons. The electrical energy is expended to transfer heat and thus to create a temperature difference between the two ends of the cylinder. The direction of the transfer of heat depends on the direction of rotation, and thus the engine can heat as well as cool the round end of the glass tube, relative to the other end that has a large metal heat reservoir attached to it, which is in turn exchanging heat with the surrounding air.

The heat pump system consists of the following parts (please refer to Fig. 3.1 for Stirling engine parts):

1. a DC power supply that is used to vary the rotations per minute (RPM) of the motor by varying the applied voltage;

2. a glass displacer piston that pumps air between the hot and cold sides of the engine. The piston is enclosed by a glass tube with a brass socket at each end; these are the temperature monitoring points of the engine;

3. a brass working piston that compresses the gas or is pushed by the expanding gas, making the conversion between mechanical and heat energy;

4. a driveshaft with balancing cams and linkages that converts to rotary motion the reciprocating motion of the working piston and also drives the displacer piston;

5. a flywheel on the driveshaft that provides rotational inertia to couple different parts of the Stirling cycle and to maintain a consistent rotational speed through the cyclical variations in the gas pressure inside the engine;

6. a direct current (DC) motor/generator that drives the engine flywheel by means of a rubber belt.

The operation of the engine is monitored with a variety of sensors. This data is sent to the PhysicaLab software and is displayed as a pV (pressure-volume) diagram. The work done per cycle, engine RPM, and the power output of the heat pump are also measured and reported by the software. Here is a summary of the sensors:

- the absolute pressure p inside the engine is monitored with a pressure sensor that outputs a reading in Pascals. This sensor typically has a resolution better than 1% of the range of pressure experienced by the engine;
- the temperatures T_1 and T_2 at each end of the displacer cylinder are measured in °C;
- the volume change V, in m^3, is calculated from a series of markings on the flywheel of the engine that determine the angle of rotation relative to a reference mark that represents the start of a pV cycle when the volume is at a maximum.

Part 1: Coefficient of performance

In this exercise you will estimate the coefficient of performance of the stirling engine heat pump. As the heat pump requires some time to reach a new operating equilibrium after a change in RPM is made, you should pay close attention to the behaviour of the engine during this time. You should document how the work done per cycle and engine power values change during these transitional periods after a change in RPM is made; you should also note and explain any systematic changes in these variables as the engine RPM is increased from a manimum to the maximum. You will be required to discuss these issues as part of your discussion.

- Review and identify all the parts of the apparatus in front of you. Rotate the engine slowly by turning the flywheel clockwise abd then counterclockwise and note the relative movement of the working piston to the displacer piston. The two pistons are offset by 90°. When does the working piston lead the displacer piston?

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- Connect the two wires between the DC electric motor and the power supply, matching the colors at the two ends. The voltage that is applied to the motor is adjusted by turning a knob and monitored on the corresponding digital display in volts (V). The current flowing through the motor is shown on the other meter in Amperes (A). Be sure that the current limit knob is turned fully clockwise.
- Turn on the power supply and increase the voltage to around 5.5 V. The engins should start to rotate and the 7-segment display should show the current engine RPM. Lets define rotation as CW

when the flywheel rotates clockwise as viewed from the side with the markings; CCW defines a counter-clockwise rotation. In which direction is the heat pump turning?

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- Start the PhysicaLab software by typing *physicalab*, followed by the Enter key. Click on the **SE** input to select the Stirling Engine interface. Press the **Get data** button to begin graphing pV cycles. In this mode, the points/s and count values are ignored; the graph updates at a rate that is determined by the speed of the engine.
- Let the engine equilibrate for several minutes, until the RPM, work and power values become steady. Press the **Stop** to temporarily freeze the graph display. Look at the data set and the oval display of the corresponding pV cycle. Where does the pV cycle begin? Is the pV cycle being drawn in a clockwise or counter-clockwise direction?

  Feel the ends of the displacer cylinder, near the brass plugs. Do you note a temperature difference? To which side is heat being pumped?

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V (V)	I (A)	P_m (W)	RPM	W (J)	P_{se} (W)	COP

Table 3.1: Stirling heat pump data for part 1

- Record in Table 3.1 the V and I for the motor displayed on the power supply, the heat pump RPM and work/cycle from the graph. Calculate the electrical power P_m used by the motor and the power P_{se} of the engine at the current RPM. Determine the COP for the heat pump at the current RPM: $COP = P_{se}/P_m$. Estimate the errors on all measurements by observing the fluctuations in each reading over a span of 30 seconds. The error can be estimated as one half of the difference between the maximum and minimum values observed.

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- Restart the graph update. Increase the voltage by 1‘V and repeat the equilibration and data recording procedure up to a voltage of 12.5 V. When the table is complete, terminate the transmission of data.
- You will now plot a curve representing the COP for the heat pump as a function of RPM. Enter your values for COP and RPM in the data window, select **Scatter plot** then press **Draw**. How does the COP vary with RPM? Is there a value of RPM where the COP seens to be a maximum?

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Part 2: Coefficient of performance, opposite direction

V (V)	I (A)	P_m (W)	RPM	W (J)	P_{se} (W)	COP

Table 3.2: Stirling heat pump data for part 2

- Turn off the power supply, then switch the two wires. Turn on the power supply and set the voltage to 5.5 V as before. The heat pump should begin to rotate in the opposite direction. Repeat the above steps and record your data in Table 3.2. Make notes below of all the observations, as in the previous section.

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IMPORTANT: BEFORE LEAVING THE LAB, HAVE A T.A. INITIAL YOUR WORKBOOK!

Discussion

To complete this lab, submit a typed or neatly written Discussion of the results of the experiment (double-spaced, 400 to 600 words). Attach your computer printouts and worksheets to this Discussion. Discuss the following issues as part of your Discussion; as always this is not a complete list:

Summarize, referring to your observations, the behaviour of the heat pump corresponding to the two directions of rotation, CW and CCW.

Did you note any variation in the amount of work done by the heat pump per cycle as RPM changed? Can you explain why this might be so? Did the power output of the heat pump vary with RPM? What did you observe as the direction of rotation of the heat pump was changed?

Use Fig. 3.2 and your observations to briefly explain the operation of the heat pump in both directions of rotation. You should describe at which part of the cycle the work done by the electric motor is converted to heat by the engine and why this causes one side of the engine to become warmer and the other side to become cooler.

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Experiment 4

Refraction of light

The phenomenon of refraction can be explained geometrically with the aid of Figure 4.1. A beam of light incident on a boundary surface is composed of wavefronts that are perpendicular to the direction of propagation of the beam. This beam will propagate more slowly through a dense medium than it does through air. If the incident beam is not normal to the boundary surface, one edge of the wavefront will enter the denser material first and be slowed down. This effect will propagate across the wavefront, changing the direction of the refracted beam relative to the incident beam. This change in direction is always toward the normal to the boundary surface when the light beam crosses into a denser medium. It will be the opposite for a beam crossing into a less dense medium.

The mathematical relationship between the incident angle i and the refracted angle r of the light beam is given by Snell's Law:

$$\frac{\sin i}{\sin r} = \frac{n_{\mathrm{II}}}{n_{\mathrm{I}}} \tag{4.1}$$

The angles i and r are measured from the normal or perpendicular direction to the boundary. The numbers n_I and n_{II} are characteristic of each medium, and are called the refractive indices. For a vacuum $n = 1$, the refractive index of air is approximately unity, and *all* other materials have $n > 1$. The refractive index is also a function of the wavelength of the light, therefore light rays of different colour have different angles r for identical angles i. This effect is called the *dispersion of light*. Note that when a ray travels from a medium to a more dense medium (e.g. air to plastic), it always refracts *towards* the normal ($r \leq i$). Conversely, when a ray travels from a medium to a less dense medium (e.g. plastic to air), it always refracts *away* from the normal ($r \geq i$).

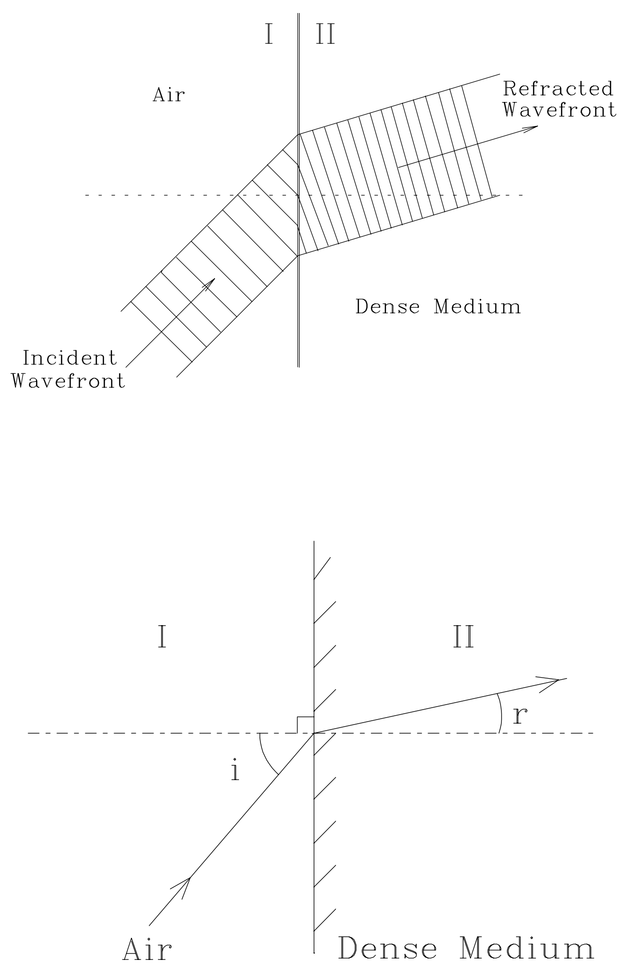

Figure 4.1: Geometry of refraction

Review questions

- **Describe** why a semicircular prism is used in Parts I - III of this experiment. How must this prism be aligned with the protractor for the measurements to be valid?

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- **Derive** the error equation for $\sigma(n)$ in Part IV of this experiment. Begin with Equation 4.4, letting $a = (1/2)(\alpha+\delta_m)$ and $b = (1/2)\alpha$. Then according to the error propagation rules, $\sigma(a) = (1/2)(\sigma(\alpha)+\sigma(\delta_m))$ and $\sigma(b) = (1/2)\sigma(\alpha)$ where $\sigma(\alpha)$ and $\sigma(\delta_m)$ are the measurement errors in α and δ_m. respectively. Recall also that $\sigma(\sin(x)) = (1/2) \mid \sin(x+\sigma(x)) - \sin(x-\sigma(x)) \mid$. **Show a complete step by step solution**.

$$n = \frac{\sin\frac{1}{2}(\alpha+\delta_m)}{\sin\frac{1}{2}\alpha}.$$

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CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Part I

You will verify Snell's Law using a semi-circular plastic prism as the second medium, as shown in Figure 4.2. Arrange the prism so that it is *concentric* with the paper protractor, with the flat surface lined up with the 90° line so that the normal (0°) is perpendicular to the flat face of the prism.

Since the incident beam of light has a finite width, the same edge of the beam should be used to set the incident angle i and to measure the refracted angle r. Make sure that this edge of the beam passes through the centre point of the protractor, otherwise your angle measurements will be incorrect.

- Vary the incident angles i by rotating the prism/protractor combination, in 10° increments from 10° to 80°, and measure the corresponding refracted angle r values. Enter your results in Table 4.1.
- Calculate the values of $\sin i$ for the incident angles i and $\sin r$ for the refracted angles r. Enter the results in Table 4.1.

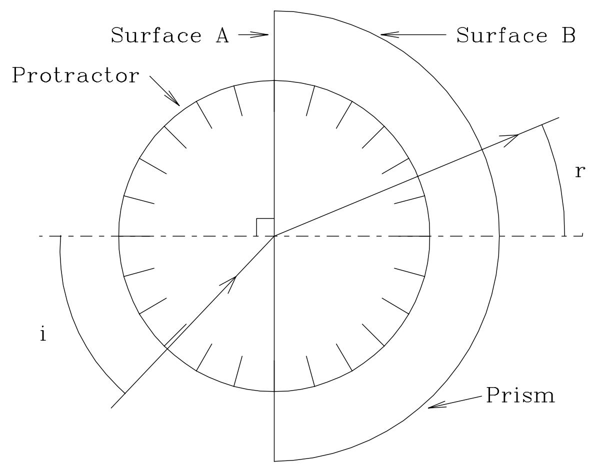

Figure 4.2: Refraction into a denser medium

i°	10	20	30	40	50	60	70	80
r°								
$\sin i$								
$\sin r$								

Table 4.1: Exprimental results for Part I

Equation 4.1 can be rearranged to give $\sin i = (n_{II}/n_I)\sin r$. This is the equation of a line with slope (n_{II}/n_I). Since in our case the incident medium is air, with index of refraction $n_I \approx 1$, a plot of $\sin i$ as a function of $\sin r$ will yield a line of slope n_{II}.

- Using the Physicalab software, enter the pairs of values $(\sin r, \sin i)$ in the data window. Select **scatter plot**. Click **Draw** to generate a graph of your data. The displayed data should approximate a straight line. Select **fit to: y=** and enter **A*x+B** in the fitting equation box. Click **Draw** to fit a straight line to your data. Label the axes and title the graph with your name and a description of the data being graphed. Click **Print** to generate a hard copy of your graph.
- Record the values for the slope and the standard deviation of the slope displayed in the printout.

$$n_{II} = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

Part II

The refractive index n for the plastic was determined in Part I using light travelling from air to plastic. Is n for the plastic prism the same if it is measured using a light ray passing from plastic to air? To answer this question, place the light source on the opposite side of the prism, as shown in Figure 4.3. Aim the beam of light so that it passes through the curved prism face (surface B). Make sure that you use the side of the beam that is closer to the normal as your reference edge and that this edge goes through the centre point of the protractor. The angle of incidence i is now measured inside the prism while angle r is outside.

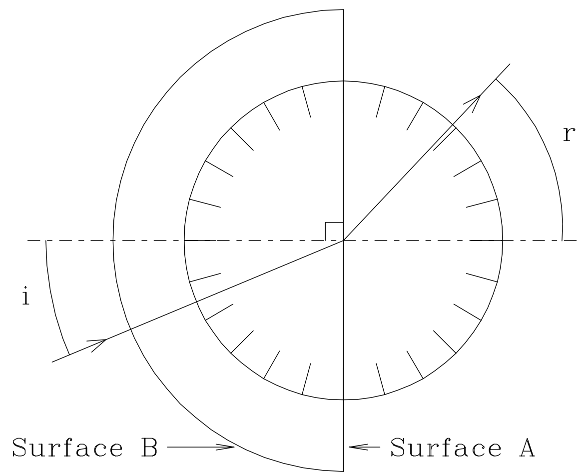

Figure 4.3: Refraction from a denser medium

- Vary the angle i of the incident beam in 5° increments from 5° to 40° and measure the corresponding values of r.
- Calculate the values of $\sin i$ for the incident angles i and $\sin r$ for the refracted angles r. Enter your results in Table 4.

$i°$	5	10	15	20	25	30	35	40
$r°$								
$\sin i$								
$\sin r$								

Table 4.2: Exprimental results for Part II

- Using the Physica Online software, graph the eight pairs of values ($\sin r$, $\sin i$). Remember that now the index of the refracted beam n_{II} is that of air, hence $N_{II} \approx 1$. With this in mind, we can conclude that the calculated value of the slope from the graph will be the inverse of the refractive index n_I of the incident medium.
- Summarize below the values for the slope and the standard deviation of the slope displayed in the fitting parameter window, and from these determine a value and error for the index of refraction of the prism n_I. Include these calculations as part of your Discussion.

$$slope = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

$$n_I = \ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

Part III

For light incident on a boundary from a denser medium, Snell's Law indicates that there is a certain angle of incidence i for which the reflected angle will be $r = 90°$. This angle of incidence is known as the *critical angle* θ_c. If $i > \theta_c$, there will be no refracted beam and the incident ray will exhibit a total internal reflection from the boundary, into the denser medium. For this special case, Snell's Law may be written as:

$$n = \frac{1}{\sin\theta_c}. \tag{4.2}$$

Using the experimental setup of Part II:

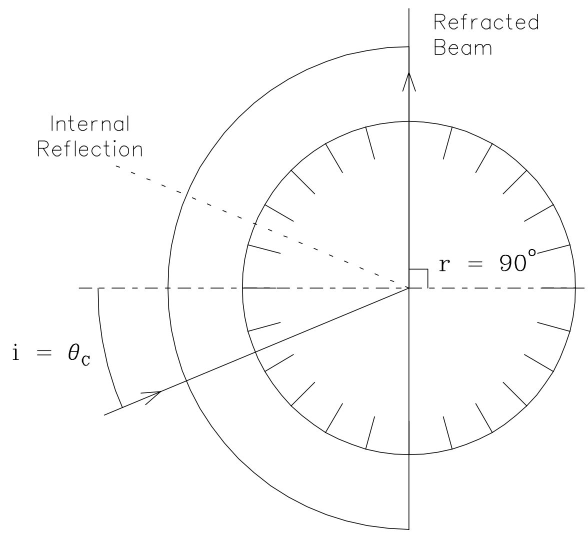

Figure 4.4: Critical angle setup for Part III

- Observe the refracted beam and adjust the angle i of the incident beam to set the angle of refraction r at 90°so that the refracted beam disappears as in Figure 4.4.
- Measure θ_c of the incident beam and choose a reasonable value for the error in θ_c. Calculate a value of n for the prism from Equation 4.2 and the error $\sigma(n)$ using the appropriate error propagation relation. Include these calculations as part of your Discussion.

θ_c = ... ± ....

n = ... ± ....

Part IV

For this part of the experiment you will use the equilateral triangle prism. Let a ray be incident on the entrance face of the prism at an angle ϕ_1, as in Figure 4.5. It will leave the prism at the exit face with an angle ϕ_2. We define the deviation angle δ as the total change in light direction, as shown in Figure 4.5.

The angle δ is given by a complicated relationship between ϕ_1, the index n, and the apex angle α of the prism. If we define δ_m as the value of the angle δ when $\phi_1 = \phi_2 = \phi$, the angle δ_m and the refractive index of the prism are given by:

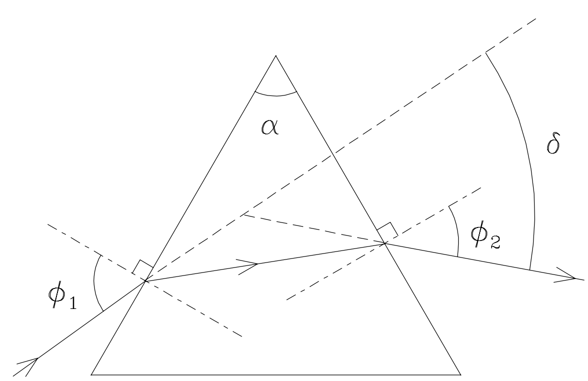

Figure 4.5: Angle of deviation, Part IV

$$\delta_m = 2\phi - \alpha, \tag{4.3}$$

$$n = \frac{\sin\frac{1}{2}(\alpha + \delta_m)}{\sin\frac{1}{2}\alpha}. \tag{4.4}$$

It can be shown that if $\phi_1 = \phi_2$, then δ_m is the minimum value of the angle δ for all values of δ. Hence, δ_m is called the *minimum angle of deviation.*

- Set up the prism and light source as shown in Figure 4.5, and use the two protractors to measure ϕ_1 and ϕ_2. Move the source around until you have ϕ_1 and ϕ_2 equal within 0.5°. Make sure that both beams are lined up properly and that they are going through the origin of each protractor.
- Measure ϕ and estimate an error for ϕ. Use Equation 4.3 to calculate a value and error for δ_m.

$$\phi = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

$$\delta_m = \ldots\ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots\ldots\ldots$$

- Use Equation 4.4 to calculate the index of refraction n and the error $\sigma(n)$ of the prism. The apex angle of the prism is $\alpha \pm \sigma(\alpha) = 60.0 \pm 0.3°$.

$$n = \frac{\sin\frac{1}{2}(\alpha + \delta_m)}{\sin\frac{1}{2}\alpha}$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$\sigma(n) = n\left[\frac{|\sin\frac{1}{2}(\alpha + \delta_m + \sigma(\alpha) + \sigma(\delta_m)) - \sin\frac{1}{2}(\alpha + \delta_m - \sigma(\alpha) - \sigma(\delta_m))|}{2\sin\frac{1}{2}(\alpha + \delta_m)}\right]$$

$$+ n\left[\frac{|\sin\frac{1}{2}(\alpha + \sigma(\alpha)) - \sin\frac{1}{2}(\alpha - \sigma(\alpha))|}{2\sin\frac{1}{2}(\alpha)}\right]$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$= \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

IMPORTANT: BEFORE LEAVING THE LAB, HAVE A T.A. INITIAL YOUR WORKBOOK!

Discussion

To complete this lab, submit a typed Discussion of the results of the experiment. Attach your computer printouts to this Discussion. Summarize your data in a table and compare your results for n obtained from the four parts of this experiment. Comment on improvements that might be made to the experimental setup and discuss the various sources of error.

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Experiment 5

Archimedes' principle

Everybody knows the story's punch line: a man is so excited by the idea that came to him in a bathtub that he runs naked to the Emperor's palace, screaming "Eureka!" But what exactly was the idea that made Archimedes forget the dress code? Survey your friends who are not taking this Physics course, and most are likely to respond with some description of *buoyancy*: a body immersed in a fluid experiences a buoyant force equal to the weight of the displaced fluid. That's important enough, and Archimedes did author a very concise formulation of the buoyancy principle. However, boats floated long before Archimedes came along. Why would a better formulation of an old idea excite the respected philosopher so?

The true story of "Eureka!" is somewhat more subtle. Archimedes figured out a way to *use* the buoyant force to solve a very important practical problem of catching the crooks who were defrauding the treasury by passing a gold-silver alloy coins as pure gold ones. Before you continue reading the next paragraph, spend a few minutes trying to think of a solution to this challenge. It's not an easy one!

Archimedes' solution (which brought him both the satisfaction of resolving a intellectual challenge and a considerable monetary reward) could be implemented quickly and easily and required only the simplest of tools: a balance scale, weights made of pure silver and pure gold, and a tub of water. First, you had to use the weights made of gold balance out the unknown material. Then you would submerge both sides of the balance in water. If the two arms remained balanced, then the unknown material was also gold.: the same mass of the material displaced the same volume of water on both sides and thus both sides experienced the same buoyant force equal to the weight of that water. If, however, the material was not really gold, its density was slightly different from that of the pure gold, and the same mass would displace a different volume of water. The buoyant force would be slightly different on the two sides of the balance scale, and the submerged balance would tilt. In fact, by replacing the pure gold weights with a mix of gold and silver weights and adjusting their ratio until the balance scale remained level in and out of water, one could measure the exact make-up of the alloy — and to catch the crooks!

In this experiment you will determine the unknown ratio of copper (Cu) to aluminum (Al) in a simulated "alloy" of the two metals, resolving essentially the same conundrum.

With the *density* of the material defined as the ratio of its mass m to its volume V, $\rho = m/V$ (in kg/m^3), and the so-called *specific gravity* of the material being the dimensionless ratio of the density of the material to that of water,

$$\mathcal{S}_x = \frac{\rho_x}{\rho_{H_2O}} ,$$

it is easy to see that a convenient practical way to measure the specific gravity, which is an important identifying characteristic of a material, is as a ratio

$$\mathcal{S}_x = \frac{\text{weight of object in air}}{\text{apparent weight loss when submerged in water}}$$

Review questions

- Prove that the above prescription does, indeed, measure ρ_x/ρ_{H_2O}.

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- Find the density and/or the specific gravity of pure Al and pure Cu; report sources and units as appropriate.

Al: . . . .

Cu: . . . .

- What is the most likely source of error in this experiment?

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CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Procedure and analysis

The experimental apparatus consists of a precise digital weight scale, a volumetric flask, a pipette, distilled water, a long bar of Cu, a long bar of Al, and a simulated Al/Cu "alloy" made up of two short bars of Al and a Cu.

For each of the three metals (Al, Cu and the Al/Cu "alloy") you will need to perform three separate measurements, as summarized in Fig. 5.1:

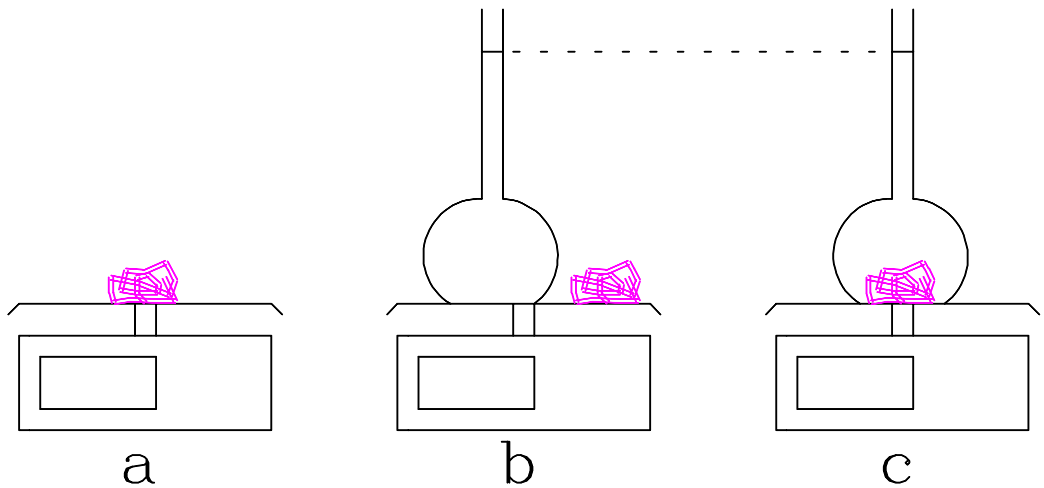

Figure 5.1: The three steps must be repeated for each metal

(**a**) weight of the piece of metal;

(**b**) weight of the piece of metal and of the volumetric task filled with distilled water to the exact mark on the neck of the flask;

(**c**) weight of the piece of metal *submerged in* the volumetric task filled with distilled water to the exact same mark.

If you do the steps (**a**)–(**c**) in order, you will need to withdraw some water from the volumetric flask between steps (**b**) and (**c**). This is best achieved by simply pouring off some water and then adding the required amount back, drop-by-drop from the pipette when the level gets close to the target mark. Be careful not to have any water droplets on the outside of the flask or on the scale platform itself.

A major source of error in this experiment is the precision with which you can reproduce the exact same water level time after time. You may want to do it several times for each measurement, pouring off a small amount of water and topping it back up to the mark, and take the average of several measurements.

You do not need to take the previous metal out of the flask when you switch to the next one. However, if you choose to do so, or if you try to re-measure the weight of the piece of metal alone, be sure to dry it thoroughly first.

- Fill in the following table of weights (record the measurement errors, too):

	Metal	Metal & Flask	Metal *in* Flask	Weight loss, ΔW	$\mathcal{S}$
Al					
Cu					
"alloy"					

- Show a sample calculation for the error in ΔW and S:

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- Assuming that our "alloy" is made of a mixture of m_{Al} of Al and m_{Cu} of Cu, prove that

$$\frac{m_{\mathrm{Al}}}{m_{\mathrm{Cu}}} = -\frac{1 - \mathcal{S}_{\mathrm{alloy}}/\mathcal{S}_{\mathrm{Cu}}}{1 - \mathcal{S}_{\mathrm{alloy}}/\mathcal{S}_{\mathrm{Al}}} \tag{5.1}$$

Start with

$$\rho_{\mathrm{alloy}} = \frac{m_{\mathrm{Al}} + m_{\mathrm{Cu}}}{V_{\mathrm{Al}} + V_{\mathrm{Cu}}}$$

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- Derive the error equation for Equation 5.1 (Hint: express as a product of terms):

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- Use Equation 5.1 to calculate the experimental mass ratio $m_{\text{Al}}/m_{\text{Cu}}$ for your “alloy”:

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- Calculate the error $\Sigma(m_{\text{Al}}/m_{\text{Cu}})$ in the experimental mass ratio:

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- Determine the theoretical mass ratio of your “alloy” by weighing the two short bars:

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- Determine the error in the theoretical mass ratio:

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IMPORTANT: BEFORE LEAVING THE LAB, HAVE A T.A. INITIAL YOUR WORKBOOK!

Discussion

To complete this lab, submit a typed or neatly written Discussion of the results of the experiment (double-spaced, 200 to 400 words). Attach your computer printouts and worksheets to this Discussion. Discuss the following issues as part of your Discussion; as always this is not a complete list:

- What are the sources of error in your experiment? Was the volume determination, indeed, the main one?
- The density of the material can vary with temperature, and not maintaining materials at a constant temperature throughout the experiment may affect the results. Was this a major factor, given our methodology?
- Is there agreement between the theoretical and experimental mass ratios? Explain.

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Experiment 6

Viscosity and drag

Drag force arises when an object moves through a fluid or, equivalently, when fluid flows past an object. In general, the drag force grows larger with increased flow velocity, but viscosity is a complex phenomenon that cannot be reduced to the simple relationship "drag force is proportional to velocity".

The origin of the drag force F_d lies in the need to displace the particles of the fluid out of the way of a moving object. At low velocities the movement of the fluid is smooth (*laminar*) and the faster the object moves, the greater is the amount of fluid that has to get out of the way, giving rise to the linear relationship

$$F_d^{(viscous)} = a\eta v$$

where v is the velocity of the object relative to the fluid, η is the *coefficient of viscosity*, and a is the "size" of the object; for a sphere of radius r, $a = 6\pi r$. The larger the size, the greater is the amount of fluid that needs to get out of the way, hence it is not surprising that the relationship is linear with respect to a as well.

The story gets far more complicated for higher velocities, unusual object shapes, of unusual fluids. At higher velocities or around oddly-shaped objects the flow stops being smooth and turns *turbulent*, which requires more energy and causes the drag force to switch to the quadratic regime, where $F_d \propto v^2$,

$$F_d^{(inertial)} = S\frac{\rho_0 v^2}{2} \, .$$

Note that this expression represents an inertial rather than a viscous force, and instead of the the viscosity, η, the fluid density, ρ_0, enters the formula. S is the cross-sectional area of the moving object.

An empirical parameter called the Reynolds number determines which of the above two terms dominates:

$$\mathcal{R} = \frac{d\rho v}{\eta}$$

where for a simple sphere, the diameter $d(= 2r)$ is the obvious definition of the "size" of the particle[1]. For $\mathcal{R} < 1$, the viscous drag dominates (and thus there is no inertial coasting), while for $\mathcal{R} > 1000$ the viscous drag is completely negligible as compared to the inertial drag.

At higher velocities still, in compressible fluids one needs to account for the compression waves that get created (*e.g.* the sonic boom of a supersonic aircraft), and in incompressible fluids the surface layer of the fluid may lose contact with the rapidly moving object creating local vacuum (*e.g.* cavitation on the surface of the blades of naval propellers). Unusual fluids such as solutions of long polymers may experience flow in directions other than the direction of motion, as the entangled polymer molecules transmit the movement of the object to remote areas of the fluid and back. The fluid constrained into finite-size channels and tubes

[1] For other shapes, an equivalent "hydrodynamic radius" r_h is used instead of r

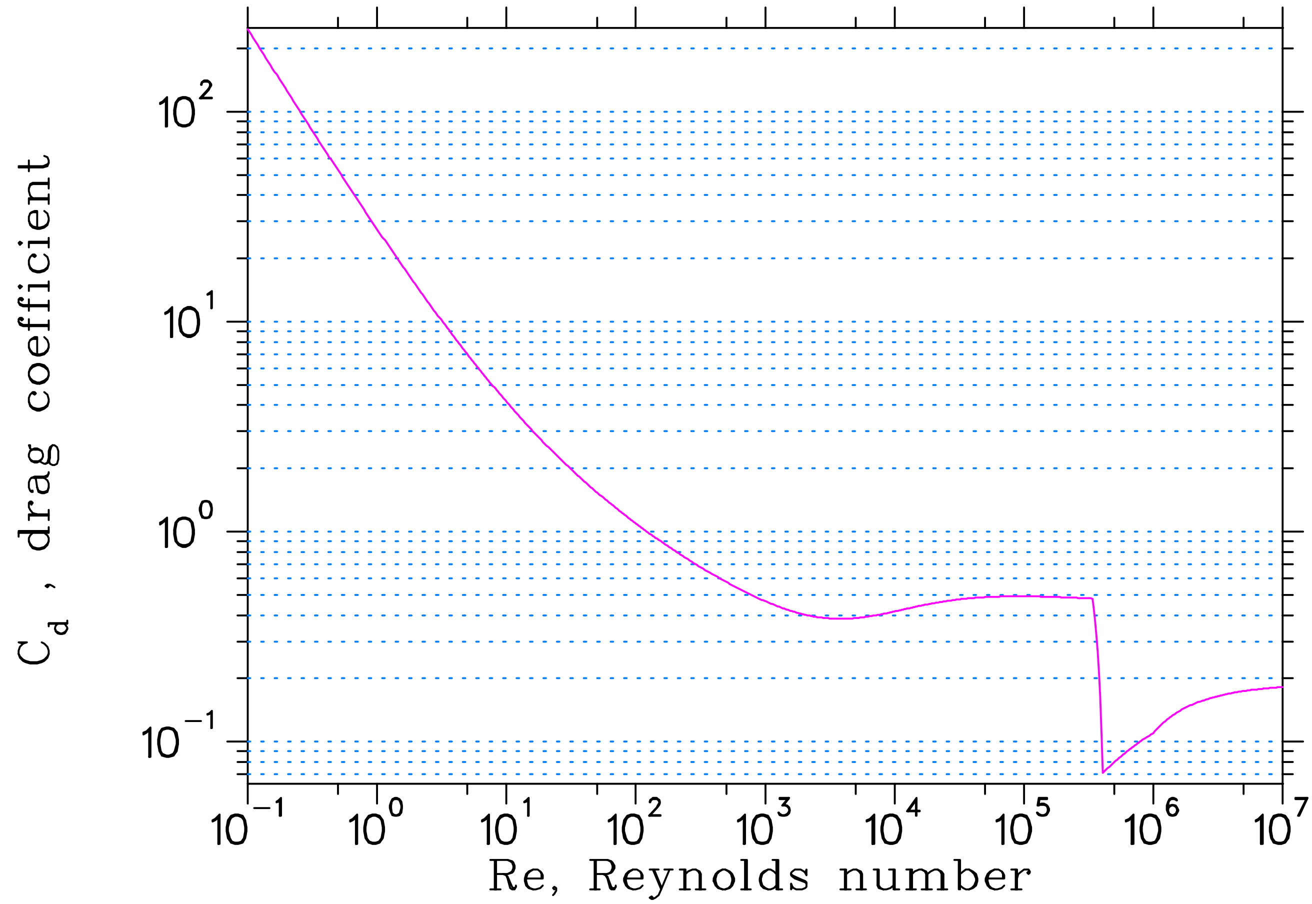

Figure 6.1: Drag coefficient on a sphere in an infinite fluid, as a function of the Reynolds number

also behaves differently from an infinitely-broad medium through which a small-size sphere is traveling. There are corrections that must account for all of these details.

A convenient way to express just such a correction is through an empirical factor called "the coefficient of drag", C_d, which can be measured experimentally and account for the details of shape, size, and the nature of the media through which an object is traveling. This dimensionless coefficient is well-known and its tabulated values can be found in handbooks. As a function of the Reynolds number, the experimentally measured values of the drag coefficient, C_d, constitute the so-called "standard drag curve" shown in Fig. 6.1. For a small sphere traveling through an infinitely broad fluid, this C_d provides an excellent agreement with the experimental drag force:

$$F_d = K_f \, C_d \, S \, \frac{\rho_0 v^2}{2}, \qquad C_d = C_d(\mathcal{R}) \tag{6.1}$$

The finite-size correction coefficient, K_f, describes how this equation is modified by the boundary conditions of a fluid confined in a finite-size vessel. For a sphere of diameter d traveling axially through a fluid channel (a pipe) of diameter D, a dimensionless ratio $x = d/D$ can be used to approximate this correction coefficient to within 6% via[2]

$$K_f = \frac{1}{1 - \alpha x^{\alpha}}, \qquad \alpha = 1.60 \text{ for } x \leq 0.6 \,, \tag{6.2}$$

in the range of Reynolds number values of $10^2 < \mathcal{R} < 10^5$.

[2]R.Clift, J.R.Grace, and M.E.Weber. *Bubbles, Drops, and Particles*, p.226. Academic Press, 1978

For an object at rest, there is no drag force. As we apply a force to an object, it experiences an acceleration, its velocity grows, and with it grows the drag force that is directed opposite the velocity. This drag force counteracts the applied force and reduces the net force and the object's acceleration. Eventually, velocity increases to the point where the drag force is exactly matched to the applied force, bringing the net force to zero. From this point on, the object is in equilibrium, there is no acceleration, and the velocity remains constant, equal to its steady-state value called *the terminal velocity*, or v_t. For example, if an object is in a free fall through a fluid, this equilibrium condition corresponds to the weight of the object exactly equal to the viscous force:

$$F_d = W \ .$$

When the fluid in question is a gas such as air, the buoyant force F_b — equal to the weight of the displaced gas — is tiny and can be neglected. When the fluid is a more dense substance such as water, the above equation may need to be modified to include the buouyant correction due to the displacement of the fluid by the object

$$F_d = W - F_b = m_{\text{object}}g - m_{\text{fluid}}g = m_{\text{object}}g(1 - \rho_0/\rho) \tag{6.3}$$

where ρ is the density of the material of the object and ρ_0 is the density of the fluid. As you can see, only the ratio of the two densities enters into the expresiion since the volumes of the object and of the fluid it displaces are the same. For example, for an Al ball in water, $\rho_0 = 1.00\times10^3$ kg/m^3 and $\rho = 2.70\times10^3$ kg/m^3.

In this experiment you will explore this relationship by varying the force and measuring the terminal velocity of several spheres of different size.

Review questions

- What are the units of the viscosity coefficient η?

  . . . .

- What is the known value of η for water at room temperature? Cite the source of your value.

  . . . .

- In terms of its diameter d, what are the cross-sectional area S and the volume V of a sphere?

  $S =$ . . . .

  $V =$ . . . .

- For a sphere of 1-cm radius moving through water at 1 m/s, calculate the Reynolds number $\mathcal{R}$. What are the dimensions of $\mathcal{R}$?

  . . . .

- For the above sphere, which drag force (inertial or viscous) dominates? What is the expected coefficient of drag, C_d?

. . . .

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CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Procedure and analysis

The experimental apparatus consists of two different-size metal spheres, a long cylindrical container filled with water, a pulley system with a weight platform and variable weights, an ultrasonic position sensor, a digital weight scale, a micrometer.

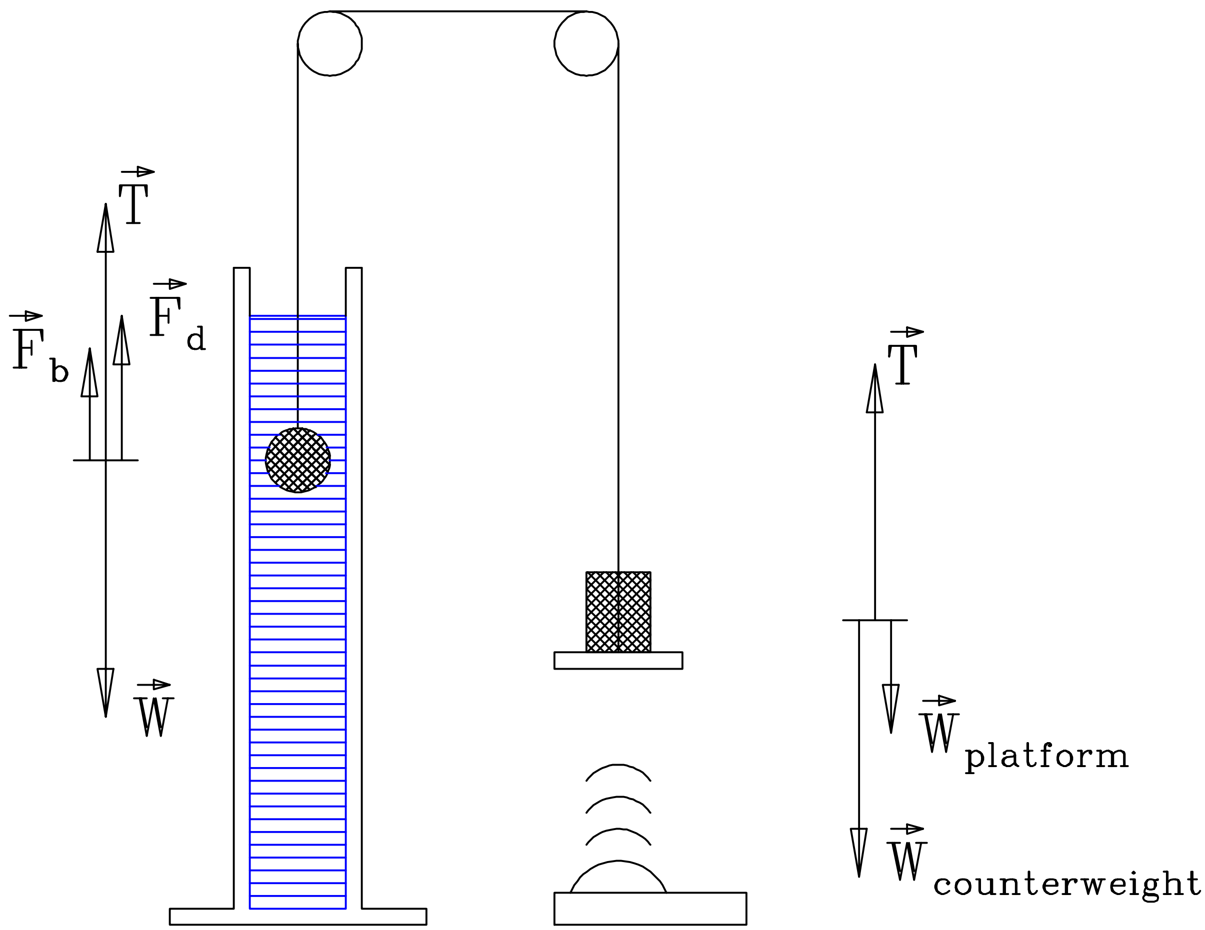

Figure 6.2: The apparatus for measuring the drag force of the fluid on a moving sphere

- Using a digital scale, weigh separately both spheres and the weight platform. You may also want to verify the exact values of the various weights in the weight set.

$W_{\text{small}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $W_{\text{large}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

$W_{\text{platform}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $W_{\text{“10 g”}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

$W_{\text{“20 g”}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $W_{\text{“50 g”}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

- Using the micrometer, measure the diameter d of each sphere and calculate their cross-sectional area $S = \pi d^2/4$.

$d_{\text{small}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $d_{\text{large}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

$S_{\text{small}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $S_{\text{large}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

- Measure and record the inner diameter D of the plastic cylinder filled with water. Verify that the cross-section is circular by measuring the diameter in a few different directions across the cylinder.

$$D = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$$

- Calculate the value of the parameter $x = d/D$ for both spheres.

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

- Select one of the spheres, adjust the length of the string so that the weight platform is at around 30 cm above the ultrasonic detector at its lowest position, and a few cm below the pulley at its highest position (*i.e.* when the sphere hits the bottom of the cylinder).
- Determine the range of masses, in 1g resolution, for which the ball does not move and hence $v = 0$. This result is related to the drag of the pulley system, and will typically span a 2-3 g range. Then start with a value of $m = 0$ and increase m to get at least three data points in each direction of motion about the region where $v = 0$.
- Shift focus to the Physicalab software. Check the **Dig1** input channel to acquire data every 0.025s, then select **scatter plot**. For each mass m, record a trace of position as a function of time as you let go of the string. If the ball flutters or wanders erratically, repeat the run. If the sphere does not reach terminal velocity, the position-time graph will not be straight near the end, omit these data

sets. Yuor graph should display a curved region when the ball is accelerating, followed by a linear region near the end of the motion when the ball is moving at a constant speed, followed by a sudden stop. Note that the sign of the slope, and thus of the terminal velocity that you record, changes as the sphere switches directions of travel. You do not need to print graphs of all these trials, but do include a couple as part of your lab report.

m, pull mass	v_t, small sphere	m, pull mass	v_t, large sphere

- For each data set, identify the region of terminal velocity just before the ball stops and the position-time graph is a straight line. Fit *only this range* of data to `A*(x-B)*(x>`X_{min}`)*(x<`X_{max}`)`, the equatin of a straight line where X_{min} and X_{max} are two numbers selected by you from a visual examination of the plot. To get a correct error value, you must also enter the constraint equation `(x>`X_{min}`)*(x<`X_{max}`)` in the constraint box. Record the pull mass used, m, as a function of the measured terminal velocity, v_t.

- From the free-body diagram of Fig.6.2, drawn for the case of the sphere traveling downward as appropriate for the small pull mass of the counterweight, when $v = v_t =$ const,

$$+T - W_{\text{counterweight}} - W_{\text{platform}} = 0$$

and thus

$$F_d + F_b + W_{\text{counterweight}} + W_{\text{platform}} - W_{\text{sphere}} = 0\,.$$

Denoting the pull mass of the counterweight as m, the mass of the weight platform as m_p, the mass of the sphere as m_s, and using the expressions of Eqs. 6.1 and 6.3 we obtain

$$K_f\, C_d\, S\, \frac{\rho_0 v_t^2}{2} = m_s g \left(1 - \frac{\rho_0}{\rho}\right) - m_p g - mg$$

Dividing through by g we obtain

$$m = m_0 - \frac{K_f C_d S \rho_0}{2g}\, v_t^2\,,$$

where $m_0 = m_s(1 - \rho_0/\rho) - m_p$.

- Repeat for the sphere traveling upward; you need to re-draw the free-body diagram of Fig.6.2 as appropriate for this case. Remember that the drag force is always opposite to the velocity of the object moving through the fluid.

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. . . .

- Enter the values obtained in the previous steps into Physicalab and plot m *vs.* v_t. Fit to a quadratic expression, `A+B*sign(x*x,x)` (the function `sign()` here accounts for a change in the direction of the drag force as v_t changes sign). Print a graph and tabulate your result for B.

- You may notice that the fit looks forced through your data points in the region near zero. This is because the fit does not take into account the region over several grams of mass where $v = 0$ as you previously noted. To observe this effect of drag on the system, repeat the previous fit, using `(A+B*x**2)*(x<0)+((A-C)-B*x**2)*(x>0)`. The fit parameter C compensates for the offset in the data set and hence this value represents the the approximate drag of the pulley system.

- Repeat the procedure for both spheres, then compare for each sphere the mass difference obtained from the graphs with that obtained previously by varying the masses:

. . . .

. . . .

- Evaluate below an error equation $\sigma(K_F)$ for the parameter K_F

$$K_F =$$

. . . .

. . . .

. . . .

. . . .

. . . .

. . . .

- Calculate K_F and the associated error for both spheres and tabulate the results.

. . . .

. . . .

- Rearrange the B term to solve for C_d, then derive $\sigma(C_d)$.

$B =$ $C_d =$

. . . .

. . . .

. . . .

	B±ΔB	$K_F \pm 6\%$	$C_d \pm \Delta C_d$
small sphere			
large sphere			
average	—	—	

- Calculate C_d and the associated error for both spheres and tabulate the results.

. . . .

. . . .

IMPORTANT: BEFORE LEAVING THE LAB, HAVE A T.A. INITIAL YOUR WORKBOOK!

Discussion

To complete this lab, submit a typed or neatly written Discussion of the results of the experiment (double-spaced, 200 to 400 words). Attach your computer printouts and worksheets to this Discussion. Discuss the following issues as part of your Discussion; as always this is not a complete list:

- What is the physical meaning of the fit parameter `A` ?
- Is there any evidence of an additional force due to the friction in the pulleys? Do your graphical and measured drag results for each ball agree? Explain. Should the drag results of the two different ball agree? Explain.
- Corrected for the finite size of the water pipe (K_F is different for the two spheres), the drag coefficient you measure should be the same in both cases. Do your two results agree? Compare the averaged C_d with the standard drag curve of Fig. 6.1 and comment on any discrepancies.

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Appendix A

Review of math basics

Fractions

$$\frac{a}{c} + \frac{b}{d} = \frac{ad + bc}{cd} \; ; \qquad \text{If} \quad \frac{a}{c} = \frac{b}{d} \, , \quad \text{then} \quad ad = cb \quad \text{and} \quad \frac{ad}{bc} = 1.$$

Quadratic equations

Squaring a binomial: $(a+b)^2 = a^2 + 2ab + b^2$
Difference of squares: $a^2 - b^2 = (a+b)(a-b)$

The two roots of a quadratic equation $ax^2 + bx + c = 0$ are given by $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Exponentiation

$$(a^x)(a^y) = a^{(x+y)} \, , \qquad \frac{a^x}{a^y} = a^{x-y} \, , \qquad a^{1/x} = \sqrt[x]{a} \, , \qquad a^{-x} = \frac{1}{a^x} \, , \qquad (a^x)^y = a^{(xy)}$$

Logarithms

Given that $a^x = N$, then the logarithm to the *base a* of a number N is given by $\log_a N = x$.

For the decimal number system where the base of 10 applies, $\log_{10} N \equiv \log N$ and

$$\begin{aligned} \log 1 &= 0 \;\; (10^0 = 1) \\ \log 10 &= 1 \;\; (10^1 = 10) \\ \log 1000 &= 3 \;\; (10^3 = 1000) \end{aligned}$$

Addition and subtraction of logarithms

Given a and b where $a, b > 0$: The log of the product of two numbers is equal to the sum of the individual logarithms, and the log of the quotient of two numbers is equal to the difference between the individual logarithms .

$$\begin{aligned} \log(ab) &= \log a + \log b \\ \log\left(\frac{a}{b}\right) &= \log a - \log b \end{aligned}$$

The following relation holds true for all logarithms:

$$\log a^n = n \log a$$

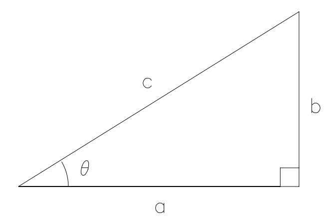
Natural logarithms

It is not necessary to use a whole number for the logarithmic base. A system based on "e" is often used. Logarithms using this base $\log_e$ are written as "ln", pronounced "lawn", and are referred to as *natural logarithms*. This particular base is used because many natural processes are readily expressed as functions of natural logarithms, i.e. as powers of e. The number e is the sum of the infinite series (with $0! \equiv 1$):

$$e = \sum_{n=0}^{\infty} \frac{1}{n!} = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots = 2.71828\ldots$$

Trigonometry

Pythagoras' Theorem states that for a right-angled triangle $c^2 = a^2 + b^2$. Defining a trigonometric identity as the ratio of two sides of the triangle, there will be six possible combinations:



$$\sin\theta = \frac{b}{c} \qquad \cos\theta = \frac{a}{c} \qquad \tan\theta = \frac{b}{a} = \frac{\sin\theta}{\cos\theta}$$

$$\csc\theta = \frac{c}{b} \qquad \sec\theta = \frac{c}{a} \qquad \cot\theta = \frac{a}{b} = \frac{\cos\theta}{\sin\theta}$$

$$\sin(\theta \pm \phi) = \sin\theta\ \cos\phi \pm \cos\theta \sin\phi \qquad \sin 2\theta = 2\sin\theta\ \cos\theta \qquad 180^\circ = \pi \text{ radians} = 3.15159\ldots$$

$$\cos(\theta \pm \phi) = \cos\theta\ \cos\phi \mp \sin\theta \sin\phi \qquad \cos 2\theta = 1 - 2\sin^2\theta \qquad 1 \text{ radian} = 57.296\ldots^\circ$$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\ \tan\phi} \qquad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} \qquad \sin^2\theta + \cos^2\theta = 1$$

To determine what angle a ratio of sides represents, calculate the inverse of the trig identity:

$$if \quad \sin\theta = \frac{b}{c}, \quad then \quad \theta = \arcsin\left(\frac{b}{c}\right)$$

For *any* triangle with angles A, B, C respectively opposite the sides a, b, c:

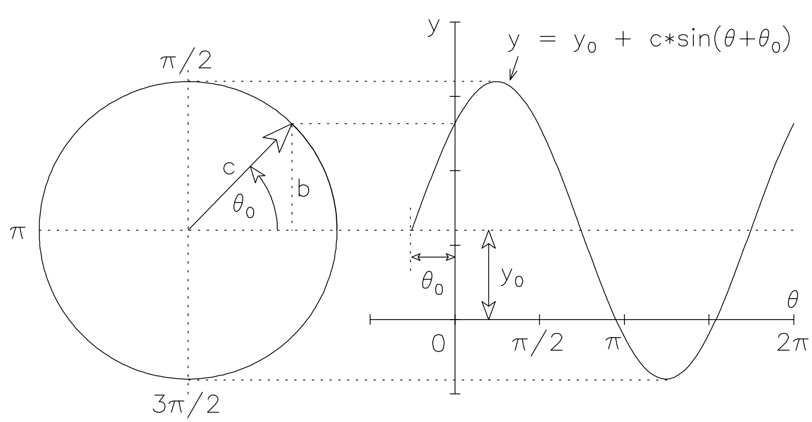
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\ ,\ (sine\ law) \qquad c^2 = a^2 + b^2 - 2ac\ \cos C.\ (cosine\ law)$$

The sine waveform

If we increase θ at a constant rate from 0 to 2π radians and plot the magnitude of the line segment $b = c\ \sin\theta$ as a function of θ, a sine wave of *amplitude* c and *period* of 2π radians is generated.

Relative to some arbitrary coordinate system, the *origin* of this sine wave is located at a *offset distance* y_0 from the horizontal axis and at a *phase angle* of θ_0 from the vertical axis. The sine wave referenced from this (θ, y) coordinate system is given by the equation

$$y = y_0 + c\sin(\theta + \theta_0)$$



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Appendix B

Error propagation rules

- The *Absolute Error* of a quantity Z is given by $\sigma(Z)$, always ≥ 0.
- The *Relative Error* of a quantity Z is given by $\frac{\sigma(Z)}{|Z|}$, always ≥ 0.
- To determine the error in a quantity Z that is the *sum* of other quantities, you *add the absolute errors* of those quantities (Rules 2,3 below). To determine the error in a quantity Z that is the *product* of other quantities, you *add the relative errors* of those quantities (Rules 4,5 below).

	Relation	**Error**	
1.	$Z = cA$	$\sigma(Z) = \lvert c\rvert\sigma(A)$	(Use only if A is a *single* term, *i.e.* $Z = 3x$.)
2.	$Z = A + B + C + \cdots$	$\sigma(Z) = \sigma(A) + \sigma(B) + \sigma(C) + \cdots$	
3.	$Z = A - B - C - \cdots$	$\sigma(Z) = \sigma(A) + \sigma(B) + \sigma(C) + \cdots$	(Error terms are *always* added.)
4.	$Z = A \times B \times C \times \cdots$	$\frac{\sigma(Z)}{\lvert Z\rvert} = \frac{\sigma(A)}{\lvert A\rvert} + \frac{\sigma(B)}{\lvert B\rvert} + \frac{\sigma(C)}{\lvert C\rvert} + \cdots$	
5.	$Z = \frac{A}{B}$	$\frac{\sigma(Z)}{\lvert Z\rvert} = \frac{\sigma(A)}{\lvert A\rvert} + \frac{\sigma(B)}{\lvert B\rvert}$	
6.	$Z = A^b$	$\frac{\sigma(Z)}{\lvert Z\rvert} = \lvert b\rvert\ \frac{\sigma(A)}{\lvert A\rvert}$	(Note the absolute value of the power.)
7.	$Z = \sin A$	$\sigma(Z) = \sigma(\sin A) = \frac{\lvert \sin[A+\sigma(A)] - \sin[A-\sigma(A)]\rvert}{2}$	(Similar for $\cos A$)
8.	$Z = \log A$	$\sigma(Z) = \sigma(\log A) = \frac{\lvert \log[A+\sigma(A)] - \log[A-\sigma(A)]\rvert}{2}$	

- $a, b, c, \ldots, z$ represent constants
- $A, B, C, \ldots, Z$ represent measured or calculated quantities
- $\sigma(A), \sigma(B), \sigma(C), \ldots, \sigma(Z)$ represent the errors in $A, B, C, \ldots, Z$, respectively.

How to derive an error equation

Let's use the change of variable method to determine the error equation for the following expression:

$$y = \frac{M}{m}\sqrt{0.5\ kx\,(1 - \sin\theta)} \tag{B.1}$$

- Begin by rewriting Equation B.1 as a product of terms:

$$
\begin{aligned}
y &= M * m^{-1} * [\,0.5 * k * x * (1-\sin\theta)]^{\;1/2} && \text{(B.2)}\\
&= M * m^{-1} * 0.5^{1/2} * k^{1/2} * x^{1/2} * (1-\sin\theta)^{1/2} && \text{(B.3)}
\end{aligned}
$$

- Assign to each term in Equation B.3 a new variable name $A, B, C, \ldots$, then express v in terms of these new variables,

$$y = A * B * C * D * E * F \tag{B.4}$$

- With $\sigma(y)$ representing the error or uncertainty in the magnitude of y, the error expression for y is easily obtained by applying Rule 4 to the product of terms Equation B.4:

$$\frac{\sigma(y)}{|y|} = \frac{\sigma(A)}{|A|} + \frac{\sigma(B)}{|B|} + \frac{\sigma(C)}{|C|} + \frac{\sigma(D)}{|D|} + \frac{\sigma(E)}{|E|} + \frac{\sigma(F)}{|F|} \tag{B.5}$$

- Select from the table of error rules an appropriate error expression for each of these new variables as shown below. Note that F requires further simplification since there are two terms under the square root, so we equate these to a variable G:

$A = M,$	$\sigma(A) = \sigma(M)$	Rule 1
$B = m^{-1},$	$\frac{\sigma(B)}{\lvert B\rvert} = \lvert -1\rvert \, \frac{\sigma(m)}{\lvert m\rvert} = \frac{\sigma(m)}{\lvert m\rvert}$	Rule 6
$C = 0.5^{1/2},$	$\frac{\sigma(C)}{\lvert C\rvert} = \left\lvert\frac{1}{2}\right\rvert \, \frac{\sigma(0.5)}{\lvert 0.5\rvert} = 0$	since $\sigma(0.5) = 0$
$D = k^{1/2},$	$\frac{\sigma(D)}{\lvert D\rvert} = \left\lvert\frac{1}{2}\right\rvert \, \frac{\sigma(k)}{\lvert k\rvert} = \frac{\sigma(k)}{2\lvert k\rvert}$	Rule 6
$E = x^{1/2},$	$\frac{\sigma(E)}{\lvert E\rvert} = \left\lvert\frac{1}{2}\right\rvert \, \frac{\sigma(x)}{\lvert x\rvert} = \frac{\sigma(x)}{2\lvert x\rvert}$	Rule 6
$F = G^{1/2},$	$\frac{\sigma(F)}{\lvert F\rvert} = \left\lvert\frac{1}{2}\right\rvert \, \frac{\sigma(G)}{\lvert G\rvert} = \frac{\sigma(G)}{2\lvert G\rvert}$	Rule 6
$G = 1 - \sin\theta,$	$\sigma(G) = \sigma(1) + \sigma(\sin\theta) = 0 + \frac{\lvert\, \sin[\theta+\sigma(\theta)] - \sin[\theta-\sigma(\theta)]\,\rvert}{2}$	Rules 3,6

- Finally, replace the error terms into the original error Equation B.5, simplify and solve for $\sigma(y)$ by multiplying both sides of the equation with y:

$$\sigma(y) = |y| \left[\, \frac{\sigma(M)}{|M|} + \frac{\sigma(m)}{|m|} + \frac{\sigma(k)}{2|k|} + \frac{\sigma(x)}{2|x|} + \frac{|\, \sin[\theta+\sigma(\theta)] - \sin[\theta-\sigma(\theta)]\,|}{|4(1-\sin\theta)|} \,\right] \tag{B.6}$$

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Appendix C

Graphing techniques

A mathematical function $y = f(x)$ describes the one to one relationship between the value of an independent variable x and a dependent variable y. During an experiment, we analyse some relationship between two quantities by performing a series of measurements. To perform a measurement, we set some quantity x to a chosen value and measure the corresponding value of the quantity y. A measurement is thus represented by a coordinate pair of values (x, y) that defines a point on a two dimensional grid.

The technique of graphing provides a very effective method of visually displaying the relationship between two variables. By convention, the independent variable x is plotted along the horizontal axis (x-axis) and the dependent variable y is plotted along the vertical axis (y-axis) of the graph. The graph axes should be scaled so that the coordinate points (x, y) are well distributed across the graph, taking advantage of the maximum display area available. This point is especially important when results are to be extracted directly form the data presented in the graph. The graph axes *do not* have to start at zero.

Scale each axis with numbers that represent the range of values being plotted. Label each axis with the name and unit of the variable being plotted. Include a title above the graphing area that clearly describes the contents of the graph being plotted. Refer to Figure C.1 and Figure C.2.

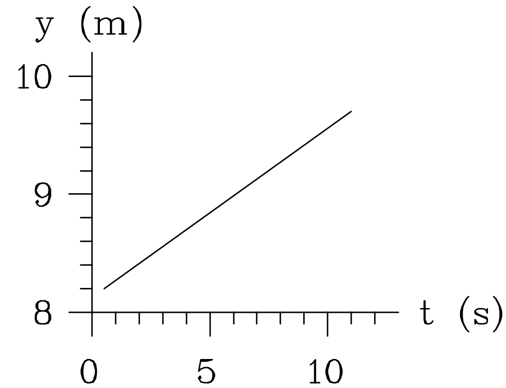

Figure C.1: Proper scaling of axes

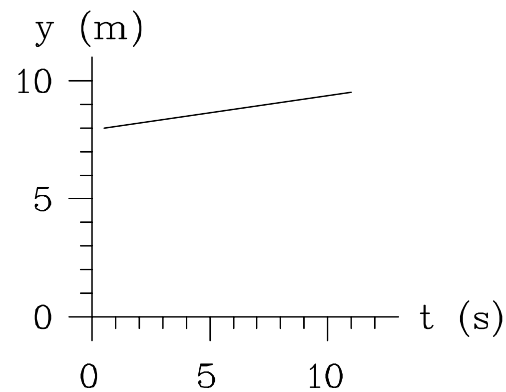

Figure C.2: Improper scaling of axes

The line of best fit

Suppose there is a linear relationship between x and y, so that $y = f(x)$ is the equation of a straight line $y = mx + b$ where m is the slope of the line and b is the value of y at $x = 0$. Having plotted the set of coordinate points (x, y) on the graph, we can now extract a value for m and b from the data presented in the graph.

Draw a line of 'best fit' through the data points. This line should approximate as well as possible the trend in your data. If there is a data point that does not fit in with the trend in the rest of the data, you should ignore it.

The slope of a straight line

The slope m of a straight-line graph is determined by choosing two points, $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, on the line of best fit, *not* from the original data, and evaluating Equation C.1. Note that these two points should be as far apart as possible.

$$m = \frac{\text{rise}}{\text{run}}$$
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \tag{C.1}$$

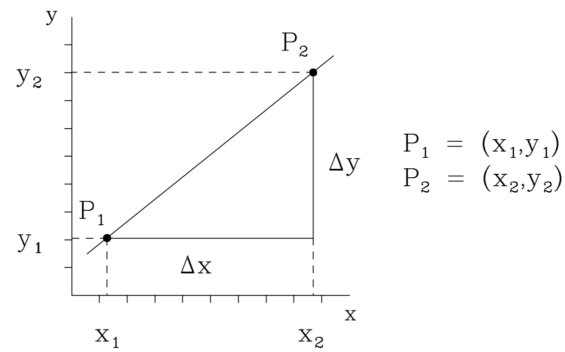

Figure C.3: Slope of a line

Error bars

All experimental values are uncertain to some degree due to the limited precision in the scales of the instruments used to set the value of x and to measure the resulting value of y. This uncertainty σ of a measurement is generally determined from the physical characteristics of the measuring instrument, i.e. the graduations of a scale. When plotting a point (x, y) on a graph, these uncertainties $\sigma(x)$ and $\sigma(y)$ in the values of x and y are indicated using error bars.

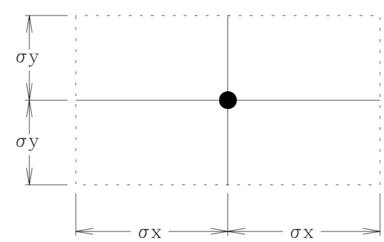

Figure C.4: Error Bars for Point (x, y)

For any experimental point $(x \pm \sigma(x), y \pm \sigma(y))$, the error bars will consist of a pair of line segments of length $2\,\sigma(x)$ and $2\,\sigma(y)$, parallel to the x and y axes respectively and centered on the point (x, y). The true value lies within the rectangle formed by using the error bars as sides. The rectangle is indicated by the dotted lines in Figure C.3. Note that only the error bars, and not the rectangle are drawn on the graph.

The uncertainty in the slope

Figure C shows a set of data points for a linear relationship. The slope is that of line 2, the line of best fit through these points. The uncertainty in this slope is taken to be one half the difference between the line of maximum slope line 1 and the line of miinimum slope, line 3:

$$\sigma(\text{slope}) = \frac{\text{slope}_{\text{max}} - \text{slope}_{\text{min}}}{2} \tag{C.2}$$

The lines of maximum and minimum slope should go through the diagonally opposed vertices of the rectangles defined by the error bars of the two endpoints of the graph, as in Figure C.

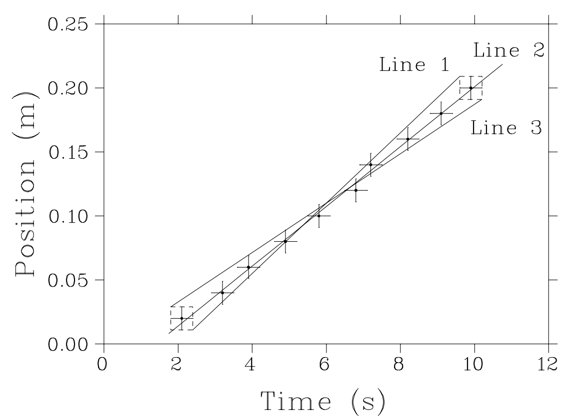

Figure C.5: Determining slope error

Logarithmic graphs

In science courses you will encounter a great number of functions and relationships, both linear and non-linear. Linear functions are distinguished by a proportional change in the value of the function with a change in value of one of the variables, and can be analyzed by plotting a graph of y versus x to obtain the slope m and vertical intercept b. Non-linear functions do not exhibit this behaviour, but can be analyzed in a similar manner with some modification. For example, a commonly occuring function is the exponential function,

$$y = ae^{bx}, \qquad \text{(C.3)}$$

where $e = 2.71828\ldots$, and a and b are constants. Plotted on linear (i.e. regular) graph paper, the function $y = ae^{bx}$ appears as in Figure C.6. Taking the natural logarithm of both sides of equation (C.3) gives

$$\begin{aligned} \ln y &= \ln\left(ae^{bx}\right) \\ \ln y &= \ln a + \ln\left(e^{bx}\right) \\ \ln y &= \ln a + bx \ln e \\ \ln y &= \ln a + bx \text{ (since } \ln e = 1) \end{aligned}$$

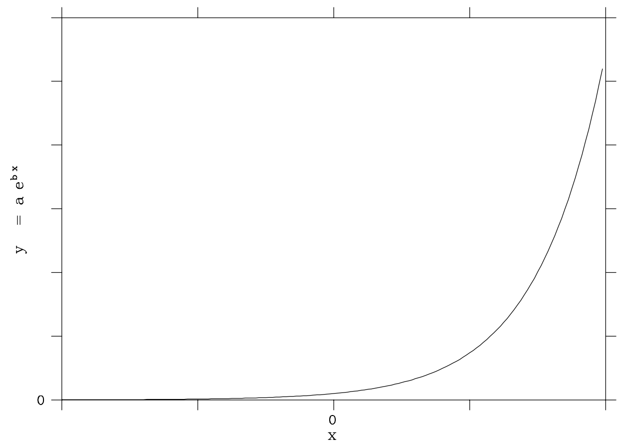

Figure C.6: The exponential function $y = ae^{bx}$.

Equation (C.4) is the equation of a straight line for a graph of $\ln y$ versus x, with $\ln a$ the vertical intercept, and b the slope. Plotting a graph of $\ln y$ versus x (semilogarithmic, i.e. logarithmic on the vertical axis only) should result in a straight line, which can be analyzed.

There are two ways to plot semilogarithmic data for analysis:

1. Calculate the natural logarithms of *all* the y values, and plot $\ln y$ versus x on linear scales. The slope and vertical intercept can then be determined after plotting the line of best fit.

2. Use semilogarithmic graph paper. On this type of paper, the divisions on the horizontal axis are proportional to the number plotted (linear), and the divisions on the vertical axis are proportional to the logarithm of the number plotted (logarithmic). This method is preferable since only the natural logarithms of the vertical coordinates used to determine the slope of the lines best fit, minimum and maximum slope need to be calculated.

Semilogarithmic graph paper

The horizontal axis is linear and the vertical axis is logarithmic. The vertical axis is divided into a series of bands called *decades* or *cycles*.

- Each decade spans one order of magnitude, and is labelled with numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 1.
- the second “1” represents 10× what the first “1” does, the third 10× the second, et cetera, and
- there is no zero on the logarithmic axis since the logarithm of zero does not exist.

A logarithmic axis often has more than one decade, each representing higher powers of 10. In Figure C.7, the axis has 3 decades representing three consecutive orders of magnitude. For instance, if the data to be plotted covered the range $1 \to 1000$, the lowest decade would represent $1 \to 10$ (divisions 1, 2, 3, ..., 9),

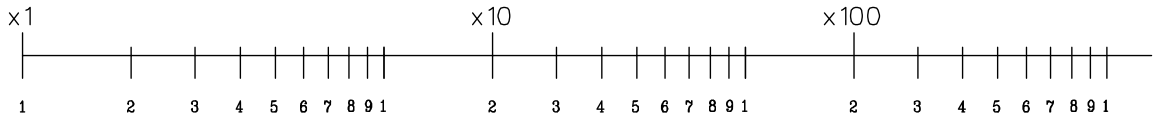

Figure C.7: A 3-Decade Logarithmic Scale.

the second decade 10 → 100 (divisions 10, 20, 30, . . . , 90) and the third decade 100 → 1000 (divisions 100, 200, 300, . . . 900, 1000).

Another advantage of using a logarithmic scale is that it allows large ranges of data to be plotted. For instance, plotting 1 → 1000 on a linear scale would result in the data in the lower range (e.g. 1 → 100) being compressed into a very small space, possibly to the point of being unreadable. On a logarithmic scale this does not occur.

Calculating the slope on semilogarithmic paper

The slope of a semilogarithmic graph is calculated in the usual manner:

$$\begin{aligned} m &= \text{slope} \\ &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\Delta(\text{vertical})}{\Delta(\text{horizontal})}. \end{aligned}$$

For Δ(vertical) it is necessary to calculate the change in the logarithm of the coordinates, not the change in the coordinates themselves. Using points (x_1, y_1) and (x_2, y_2) from a line on a semilogarithmic graph of y versus x and Equation C.4, the slope of the line is obtained.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ m &= \frac{\ln y_2 - \ln y_1}{x_2 - x_1} \\ m &= \frac{\ln (y_2/y_1)}{x_2 - x_1} \qquad \text{(C.4)} \end{aligned}$$

Note that the units for m will be (units of $x)^{-1}$ since $\ln y$ results in a pure number.

Analytical determination of slope

There are analytical methods of determining the slope m and intercept b of a straight line. The advantage of using an analytical method is that the analysis of the same data by anyone using the same analytical method will always yield the same results. Linear Regression determines the equation of a line of best fit by minimizing the total distance between the data points and the line of best fit.

To perform "Linear Regression" (LR), one can use the preprogrammed function of a scientific calculator or program a simple routine using a spreadsheet program. Based on the x and y coordinates given to it, a LR routine will return the slope m and vertical intercept b of the line of best fit as well as the uncertainties $\sigma(m)$ and $\sigma(b)$ in these values. Be aware that performing a LR analysis on non-linear data will produce meaningless results. You should first plot the data points and determine visually if a LR analysis is indeed valid.