Experiment 6

Time and frequency response of RC and RCL circuits

The measurement of transients using an oscilloscope is applied to different RC and RCL circuits. In addition to the time-domain (transient) measurements, the frequency-domain measurements can also be performed, and a transfer function of a device can be obtained.

6.1 Low-pass and high-pass filters

Additional components required

- one 10 kΩ resistor
- one 0.01 µF capacitor

We begin with the same setup as in Experiment 4. The RC circuit of Section 4.2 can be thought of as a filter with centre frequency

\[ f_0 = \frac{\omega_0}{2\pi} = \frac{1}{2\pi RC}. \]

In this Section we measure its transfer function, i.e. the relationship between the input and the output voltages.

1. Begin by determining the centre frequency for the filter, then determine for reference the transfer function and phase shift curve for the test circuit by simulating it in EWB. Include screen captures of your simulated results.

2. Make a table of test frequencies that will provide meaningful results (i.e. will define the passband region, the \( f_0 \) point and dB/octave as well as dB/decade changes in the rolloff region). Nine or ten well chosen frequencies in the range \( 0.1f_0 \) to \( 10f_0 \) should be more than sufficient to plot the gain/phase graphs.

3. Select a sinusoidal waveform on the function generator (FG), set a test frequency then measure and tabulate the signal gain \( G = \left| \frac{V_{out}}{V_{in}} \right| \). Include error estimates for all your measurements.

For the same frequency measure and tabulate the phase shift \( \phi \) in \( V_{out} \) relative to \( V_{in} \). By convention, the phase shift is positive if a point on the output signal precedes or leads in time the same point on the input signal and negative if it follows or lags in time the same point on the input signal.

You can measure the phase shift by comparing the two signals in the time domain. Begin by measuring and then recording the time delay \( \Delta T \) between two corresponding points on the input and output signals. Adjust the timebase to maximize the cursor resolution.
Determine the period of either signal $T = 1/f_0$ from the set FG frequency $f_0$ to get the phase shift. Be sure to note whether the output signal leads or lags the input signal.

You can also determine the phase shift by comparing the relative amplitudes of the two signals at the same moment in time from measurements of a Lissajous pattern on the oscilloscope screen. This method displays one signal on the Y-axis as a function of the other signal on the X-axis.

Set the time/division on the scope to X/Y mode and follow the procedure outlined in the Appendix.

Repeat the above steps for the other selected values of $f_0$. Tabulate all measurements and calculated values. Plot $\log G$ as a function of $\log f$ to generate the transfer function of the RC filter circuit. Plot $\phi$ as a function of $\log f$ to get the phase response of the circuit.

Do your graphs look similar to those obtained from the EWB simulation?

Interchange $C$ and $R$ and repeat the above steps.

Compare the two transfer functions in terms of their centre frequencies $f_0$, their decibel (dB) values at $f_0$ and rolloff rates in dB/octave. What is the order of these filters? How do the phase shifts of the two filters differ?

### 6.2 RCL transients (ringing)

**Additional components required**
- one 100 Ω resistor
- one 2.2 mH inductor

Adding an inductor $L$ to an RC circuit produces a circuit capable of resonant oscillations (ringing). The presence of $R$, an energy-dissipating element, guarantees that the amplitude of the oscillations does not remain constant. Typically, one observes oscillations of frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad [\text{rad} \cdot s^{-1}]$$

or

$$f_0 = \frac{1}{2\pi \omega_0} \quad [\text{Hz}]$$

and the amplitude of these oscillations decays exponentially with a time constant $\tau = L/R$.

Set FG to about $\frac{1}{10} f_0$. Drive the RCL circuit with a square wave and observe the ringing. Take a screen capture of the observed waveform. Measure the period of the ringing signal on the scope, and calculate the experimental value for $f_0$, including error estimates.
6.3. A BANDPASS FILTER

Compare with the theoretical value and account for any discrepancies.

The rate of energy dissipation in the circuit is determined by the time constant \( \tau = \frac{L}{R} \). Estimate the experimental value of \( \tau \) assuming an exponential envelope of the ringing signal.

Note that for an exponential function, \( y = y_0 e^{-ax} \)

\[
\frac{y_1}{y_2} = \frac{y_0 e^{-ax_1}}{y_0 e^{-ax_2}} = e^{-a(x_1-x_2)} \rightarrow \ln y_1 - \ln y_2 = -a(x_1 - x_2)
\]

\[
\sim a = \frac{\ln y_2 - \ln y_1}{x_1 - x_2}
\]

Thus one can use any two points on the exponential envelope, e.g., two peaks in the ringing signal, to determine \( a = 1/\tau \).

Compare the experimental value of \( \tau \) with the theoretical value you obtain from the nominal component values. Include error estimates.

6.3 A bandpass filter

In terms of its frequency response, a resonant \( RCL \) circuit is essentially a bandpass filter: signal frequencies near \( f_0 \) cause a large current \( I \), and hence, a large \( V_{out} = RI \), while for frequencies away from \( f_0 \) the current is small.

Switch FG to sine wave output and measure gain \( G = |V_{out}/V_{in}| \) as a function of frequency. You need to take measurements more often near \( f_0 \), and to include points up to two decades away from \( f_0 \), i.e., from \( 10^{-2} f_0 \) to \( 10^2 f_0 \). Plot \( \phi \) vs. \( \log f \), and \( G \) vs. \( \log f \).

From your graphs determine \( f_0 \) and compare to the theoretical value calculated previously. As before, include reference waveforms from the EWB simulated circuit.

6.4 A notch filter

Reconfigure the circuit as shown in Figure 6.2. Again, use \( L = 2.2 \text{ mH}, C = 0.01 \mu \text{F}, \) and \( R = 100 \Omega \). This circuit is a filter which does the reverse of the bandpass filter: it passes all but a narrow range of frequencies. A common name for this circuit is a notch filter. The notch frequency \( f_0 \) is again determined by \( L \) and \( C \), and our previously calculated theoretical value applies.

![Figure 6.2: RCL circuit as a notch filter](image)

Repeat the measurements of the previous section, i.e., plot \( G \) and \( \phi \) vs. frequency. Determine \( f_0 \) from the plot and compare to the theoretical value. Verify the frequency and phase response with Electronics Workbench.
Change capacitor to a value around $C = 0.1 \mu F$. Scan the signal frequency and determine the new notch frequency, $f_0'$. 

Verify that 

$$\frac{f_0}{f_0'} = \frac{1/\sqrt{C}}{1/\sqrt{C'}} = \frac{\sqrt{C'}}{C}.$$