

# Canadian Association of Physicists Undergraduate Prize Exam

February 2, 2010

## **Instructions**

Duration: three hours

Calculators may be used.

There are a total of 9 questions.

Write each solution on a separate page. If more than one page is required for any question, then those pages should be stapled together separate from other questions.

Please write the number of the question, your name and the name of your university clearly on the first page of each answer.

Each question has the same value. You are not expected to attempt all the questions. Relax and attempt the questions on material that you are most familiar with or those questions that look the most interesting to you.

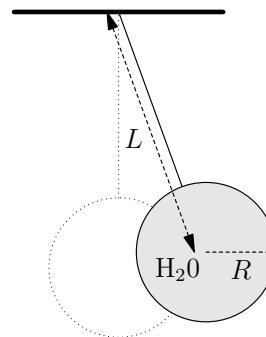
Completed exam papers should be sent by departmental organizers to:

Prof. Igor Herbut,  
Dept. of Physics,  
Simon Fraser University,  
8888 University Drive,  
Burnaby, British Columbia,  
V5A 1S6

This year's exam was prepared at Simon Fraser University by Igor Herbut, with assistance from: Malcolm Kennett, Levon Pogonian, Steve Dodge, and David Senechal (University of Sherbrooke).

**Problem 1**

The interior of a thin spherical shell of mass  $M$  and radius  $R$  is completely filled with water and hangs from a ceiling on a light thread. The distance from the sphere's center to the hanging point is  $L$ , and the mass of the water is  $m$ . Determine the change in the frequency of small amplitude oscillations of this system when the water freezes. (Neglect the viscosity of water, and the change of volume upon freezing.)

**Problem 2**

Show that a mechanical system cannot have only two components of its angular momentum conserved.

**Problem 3**

Two electrons are created in the singlet state of their total spin. If the  $z$ -component of the spin of one of the electrons is measured, in which state will the second electron be after the measurement? What is the average spin vector of the second electron in such a state?

**Problem 4**

Determine the coefficients  $\{\alpha, \beta, \gamma\}$  in the two-dimensional “Dirac equation”

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{x}, t) = (\alpha c p_1 + \beta c p_2 + \gamma m c^2) \Psi(\vec{x}, t),$$

where the momentum operator is the usual  $p_j = (\hbar/i)\partial/\partial x_j$ ,  $c$  is the velocity of light, and  $m$  is the particle's mass, so that the energy of the particle has the relativistic form  $E^2 = c^2(p_1^2 + p_2^2) + m^2 c^4$ .

**Problem 5**

The “Ising model” of ferromagnetism is defined by the configurational energy

$$E = -J \sum_{i,j} s_i s_j$$

where  $s_i = \pm 1$  is the “up” or “down” orientation of the magnetic dipole on the site “ $i$ ”,  $J > 0$  is the interaction energy between dipoles, and the sum is performed over the nearest neighbors on some regular lattice. Find the free energy of the Ising model in one dimension, by assuming that the dipoles lie on a very long chain, and show that the system does not suffer a paramagnet-ferromagnet phase transition at any finite temperature.

**Problem 6**

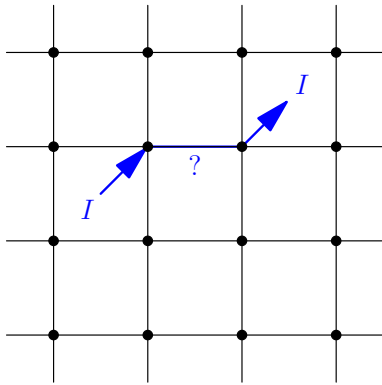
Find the dimension of space in which the Bose-Einstein condensation temperature of an ideal gas of bosons vanishes.

**Problem 7**

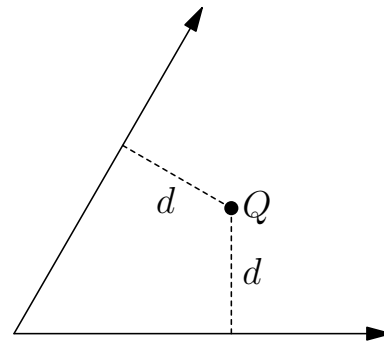
A polymer consists of  $N$  molecular units, each of length  $a$ , and is in a state of thermodynamic equilibrium at temperature  $T$ . The units are joined so as to permit a free rotation in any direction about the joints. Find the relation between the tension  $X$  acting between the two ends of the three-dimensional chain molecule and the distance between the ends. (This is the simplest model of rubber's elasticity).

**Problem 8**

An electrical circuit is made of wires connected so as to form an infinite, two-dimensional, periodic square lattice. If a current  $I$  is injected at one of the points where two wires cross (a crossing) and then extracted at any one of the closest crossings, what will be the current flowing through the wire that connects the two crossings?



(Problem 8)



(Problem 9)

**Problem 9**

Two infinite conducting planes intersect at an angle of  $60^\circ$ . A point charge  $Q$  is placed in between the planes at the same distance  $d$  from both planes. Find the electrostatic potential everywhere between the planes.