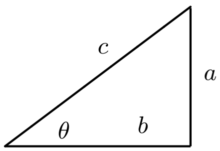
Mathematics

quadratic equation, $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

trigonometry: $a^2 + b^2 = c^2$



$$\begin{aligned} \sin\theta &= a/c \\ \cos\theta &= b/c \\ \tan\theta &= a/b \\ \tan\theta &= \frac{\sin\theta}{\cos\theta} \end{aligned}$$

circular arc (θ in rad): $s = \theta r$
for a full circle, $\theta = 2\pi$ and $s = 2\pi r$

Fundamental Constants

$g =$ 9.80 m·s^{-2}
$c =$ speed of light $= 2.99792458 \times 10^8 \approx 3.00 \times 10^8$ ms^{-1}
$m_e =$ mass of electron $= 9.11 \times 10^{-31}$ kg
$m_p =$ mass of proton $= 1.6726 \times 10^{-27}$ kg
$m_n =$ mass of neutron $= 1.6749 \times 10^{-27}$ kg
$G =$ gravit. constant $= 6.674 \times 10^{-11}$ N·m^2·kg^{-2}
$R_E =$ radius of Earth $= 6.376 \times 10^6$ m
$M_E =$ mass of Earth $= 5.9736 \times 10^{24}$ kg
$R_L =$ radius of Moon $= 1.74 \times 10^6$ m
$M_L =$ mass of Moon $= 7.35 \times 10^{22}$ kg
$T_L =$ orbital period $= 2.36 \times 10^6$ s
$R_{E-L} =$ Earth-Moon distance $= 3.84 \times 10^8$ m
$R_S =$ radius of Sun $= 6.96 \times 10^8$ m
$M_S =$ mass of Sun $= 1.99 \times 10^{30}$ kg
$R_{E-S} =$ Earth-Sun distance $= 1.496 \times 10^{11}$ m

Mechanics

motion in a straight line with $a =$ constant:

$$\begin{aligned} x &= x_0 + \bar{v}t \longrightarrow \Delta x = \bar{v}t \\ \bar{v} &= \frac{v_0 + v}{2} \\ v &= v_0 + at \longrightarrow \Delta v = at \\ \Delta x &= v_0 t + \frac{1}{2}at^2 \\ v^2 &= v_0^2 + 2a\Delta x \end{aligned}$$

Relative velocity: $\vec{\mathbf{v}}_{AC} = \vec{\mathbf{v}}_{AB} + \vec{\mathbf{v}}_{BC}$
Newton's Laws:

N1L: $\vec{\mathbf{F}}_{\text{net}} = 0 \implies \vec{\mathbf{a}} = 0$

N2L: $\vec{\mathbf{a}} = \frac{1}{m}\vec{\mathbf{F}}_{\text{net}} \longrightarrow \vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}}$

N3L: $\vec{\mathbf{F}}_{\text{A on B}} = -\vec{\mathbf{F}}_{\text{B on A}}$

Gravity: $F = G\,\frac{m\,M}{r^2}$

friction:

$$\begin{aligned} f_s &\leq f_s^{(max)} = \mu_s N \\ f_k &= \text{const} = \mu_k N, \qquad \mu_k < \mu_s \end{aligned}$$

kinematics of circular motion with $\alpha =$ constant:

$$\begin{aligned} \theta &= \theta_0 + \bar{\omega}t \longrightarrow \Delta\theta = \bar{\omega}t \\ \bar{\omega} &= \frac{\omega_0 + \omega}{2} \\ \omega &= \omega_0 + \alpha t \longrightarrow \Delta\omega = \alpha t \\ \Delta\theta &= \omega_0 t + \frac{1}{2}\alpha t^2 \\ \omega^2 &= \omega_0^2 + 2\alpha\Delta\theta \end{aligned}$$

rolling without slipping: $\Delta s = r\Delta\theta$, $v_T = r\omega$, $a_T = r\alpha$
centripetal force and acceleration (for $\alpha = 0$):

$$F_c = ma_c = m\,\frac{v_T^2}{r} = mr\omega^2$$

torque $\tau =$ force $\times$ lever arm $= Fr_\perp = rF\sin\theta$
moment of inertia: $I = mr^2$
about an axis through the geometrical centre:

$$I_{\text{hoop}} = mr^2 \quad I_{\text{disk}} = \frac{1}{2}mr^2 \quad I_{\text{rod}} = \frac{1}{12}mL^2$$
$$I_{\text{sphere}} = \frac{2}{5}mr^2 \quad I_{\text{sph.shell}} = \frac{2}{3}mr^2$$

N2L for rotations: $\tau = I\alpha$
work $W = Fd\cos\theta$, or $W = \tau\,\Delta\theta$
work done by a conservative force: $W_{\text{c.f.}} = -\Delta U$
gravitational potential energy

$$F_{\text{gravity}} = -mg \quad \implies \quad U_g = mgy$$

kinetic energy of translation: $K = \frac{1}{2}mv^2$
kinetic energy of rotation: $K = \frac{1}{2}I\omega^2$

conservation of total mechanical energy

$$\Delta U + \Delta K = W_{\text{non-c.f.}}$$

power = rate of doing work $= W/\Delta t = Fv$ or $= \tau\omega$
1 Watt = 1 J/s and 1 h.p. = 746 W
linear momentum $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
Impulse $= \bar{\vec{\mathbf{F}}}\,\Delta t = \Delta\vec{\mathbf{p}}$
conservation of momentum for a system as a whole:

$$\sum \vec{\mathbf{F}}_{\text{ext}} = 0 \implies \vec{\mathbf{p}}_{\text{total}} = \text{const}$$

angular momentum $\vec{\mathbf{L}} = I\vec{\omega}$, and $\bar{\vec{\tau}} = \frac{\Delta\vec{\mathbf{L}}}{\Delta t}$

conservation of angular momentum:

$$\vec{\tau}_{\text{net}} = 0 \implies \vec{\mathbf{L}}_{\text{total}} = \text{const} \implies I_1\omega_1 = I_2\omega_2$$

Waves and Sound

simple harmonic oscillator, $F = -kx$, $U = {}^1\!/_2\,kx^2$

$$f = 1/T \quad \omega = 2\pi f = \sqrt{k/m}$$
$$\begin{cases} x = A\cos\theta = A\cos\omega t \\ v = -A\omega\sin\omega t \\ a = -A\omega^2\cos\omega t \end{cases}$$

travelling wave $y = A\cos\left(\omega t \mp \frac{2\pi}{\lambda}x\right)$ with $v = \lambda/T = \lambda f$
string of mass m, length ℓ, volume density ρ:

$$v_{\text{string}} = \sqrt{\frac{F}{(m/\ell)}} = \sqrt{\frac{F}{(\rho A)}}$$

intensity level: $\beta = 10\,\log\left(\frac{I}{I_0}\right)$ dB, $I_0 = 1 \times 10^{-12}$ W/m^2

point source intensity: $I = \frac{P}{A} = \frac{P}{4\pi r^2}$

Doppler (upper sign = approach, lower = recede)

$$f = f_s\left(\frac{1 \pm \frac{v_o}{v}}{1 \mp \frac{v_s}{v}}\right)$$

$v =$ speed of sound
$v_s =$ speed of source
$v_o =$ speed of observer