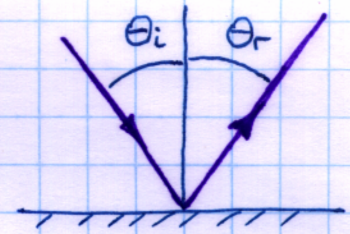
Light reflection and refraction

- law of reflection

θ_i = angle of incidence

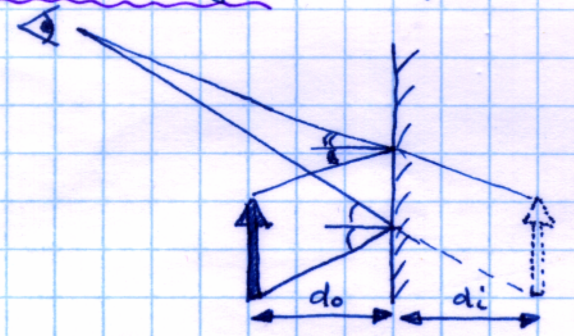
θ_r = angle of reflection

$$\boxed{\theta_i = \theta_r}$$



– specular (regular) vs. diffuse (irregular) reflection

– virtual images are formed by a plane mirror

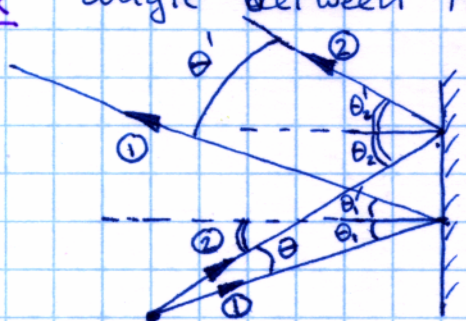


rays seem to emanate from behind the mirror, with

$$d_{object} = d_{image}.$$

or $\boxed{d_o = d_i}$

Ex angle between two rays is preserved upon reflection



incident: $\theta = \theta_1 - \theta_2$

reflected: $\theta' = \theta_1' - \theta_2' = \theta_1 - \theta_2 = \theta$

but: $\theta_1 = \theta_1'$, $\theta_2 = \theta_2'$

- refraction of light

– speed of light in a medium, v, is specified as its index of refraction, n:

$$\boxed{n \equiv \frac{c}{v}} \begin{array}{l} = \text{speed of light in vacuum} \\ = \text{speed of light in medium} \end{array}$$

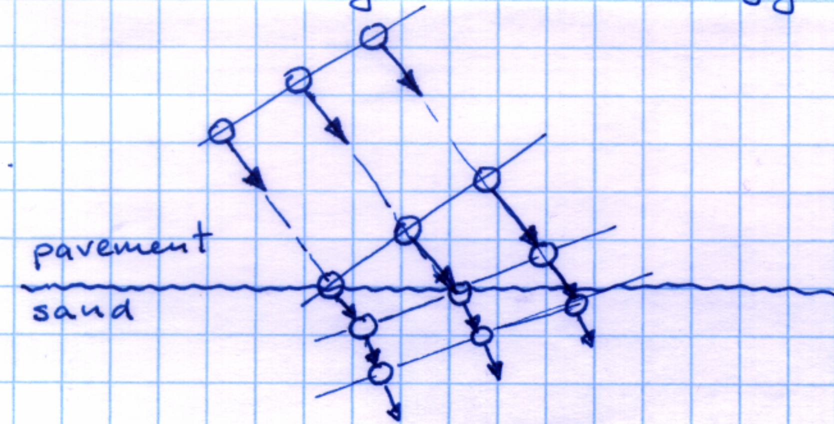
or $\boxed{v = \frac{c}{n}}$

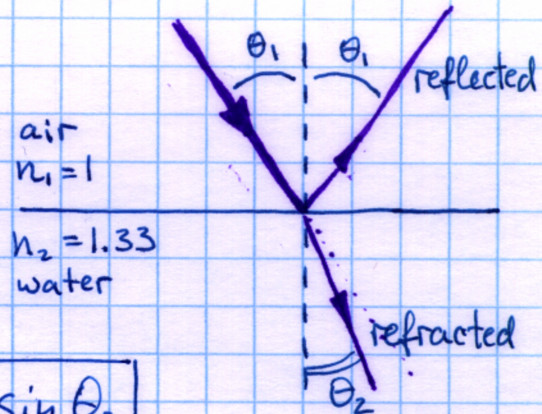
Note: $n = \frac{c}{v} = \frac{\lambda/T}{\lambda_m/T} = \frac{\lambda f}{\lambda_m f} = \frac{\lambda}{\lambda_m}$

since v always $\leq c$ $\Rightarrow$ n always ≥ 1, $\lambda_m = \frac{\lambda}{n} \leq \lambda$

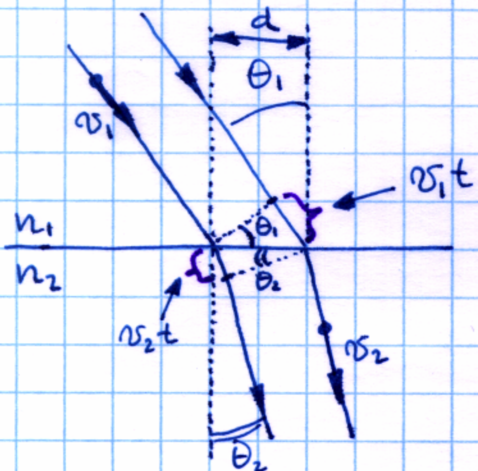
air: $n \simeq 1$

– a marching band analogy





– Snell's law $\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$

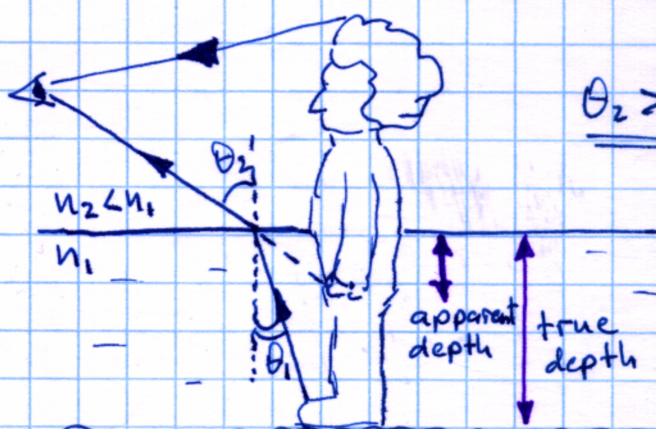


$$\sin\theta_1 = \frac{v_1 t}{d} \qquad \sin\theta_2 = \frac{v_2 t}{d}$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1 t/d}{v_2 t/d} = \frac{v_1}{v_2} = \frac{c/n_1}{c/n_2} = \frac{n_2}{n_1}$$

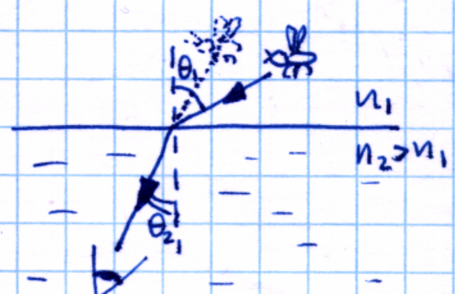
$$\Rightarrow \quad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Ex apparent depth : looking into more dense medium



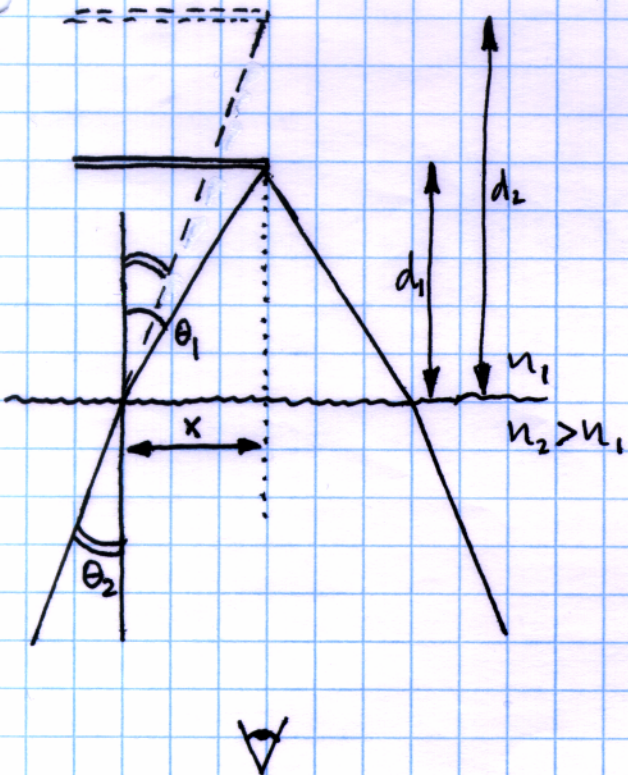
$\theta_2 > \theta_1$

Ex looking out of more dense (optically) medium :



$\theta_2 < \theta_1$

Ex height of a diving board



d_2 = apparent height = 4.0m

d_1 = actual height = ?

Snell's Law: $\dfrac{n_2}{n_1} = \dfrac{\sin\theta_1}{\sin\theta_2}$

Geometry:

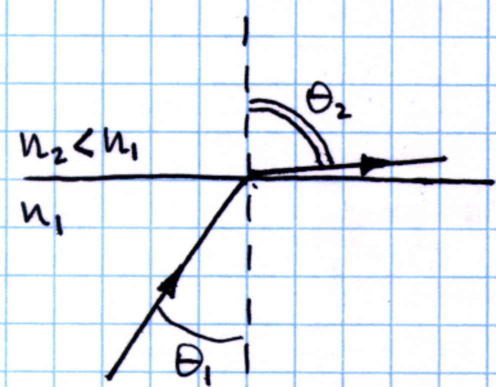
$$\sin\theta_1 = \frac{x}{\sqrt{x^2+d_1^2}} \approx \frac{x}{d_1}, \text{ for small } x$$

$$\sin\theta_2 = \frac{x}{\sqrt{x^2+d_2^2}} \approx \frac{x}{d_2}, \text{ for small } x$$

$$\Rightarrow \frac{n_2}{n_1} \approx \frac{x/d_1}{x/d_2} = \frac{d_2}{d_1}$$

$$\Rightarrow \underline{d_1 = d_2\frac{n_1}{n_2}} = 4.0\text{m}\,\frac{1.0}{1.33} \simeq \underline{3.0\text{m}}$$

- total internal reflection



If n_2/n_1 is sufficiently small, θ_2 will approach 90°:

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\cancel{\sin\theta_2}_{\to 1}} = \sin\theta_1$$

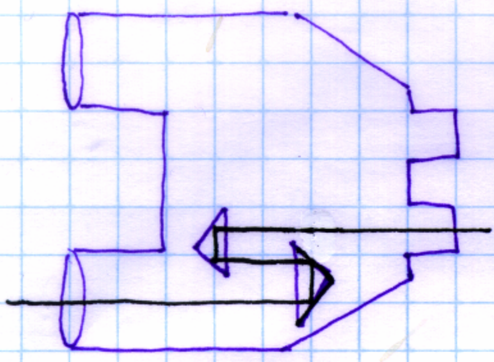
$\Rightarrow$ For all $\theta_1 > \boxed{\theta_{critical} = \arcsin\frac{n_2}{n_1}}$

there is no refracted beam, only a reflected one

$\Rightarrow$ total internal reflection

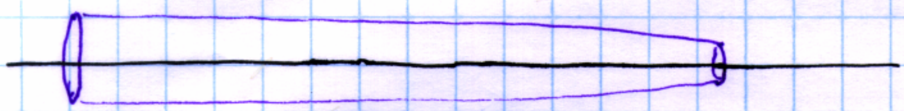
Note: from n_1 into $n_2 > n_1$, θ_2 gets smaller, i.e. no problem.

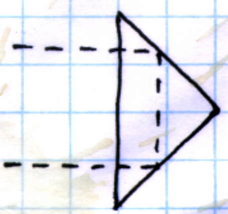
Ex prism binoculars



a "folded" optical path results in a more compact design

c.f. a telescope:





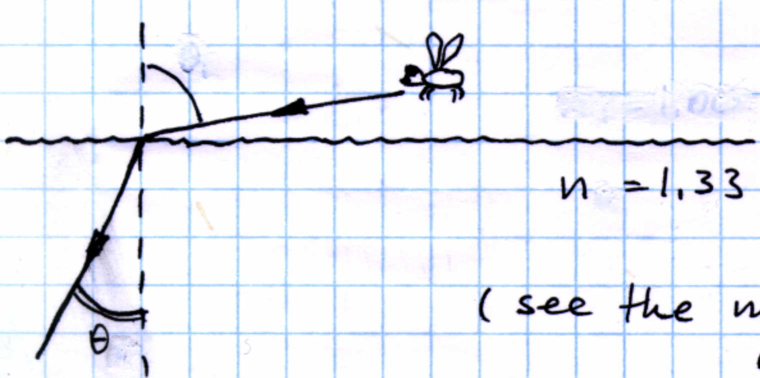
requires $\theta_1 = 45^\circ > \theta_{critical}$

$\Rightarrow \quad \frac{n_2}{n_1} < \sin 45^\circ$

$\Rightarrow \quad n_1 > \frac{n_2}{\sin 45^\circ} = \frac{n_2}{1/\sqrt{2}} = \sqrt{2}\, n_2 \approx 1.41..\, n_2$

e.g. from glass ($n_1 = 1.5$) to air ($n_2 = 1$)

Ex where is the fly?



$n = 1.33 = \frac{4}{3}$

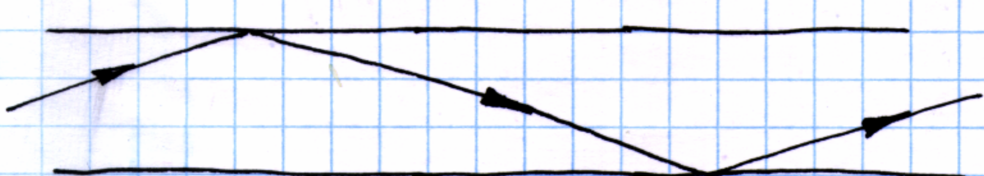
$\theta_c = \arcsin \frac{1}{n}$

$= \arcsin \frac{3}{4} = 48.6^\circ$

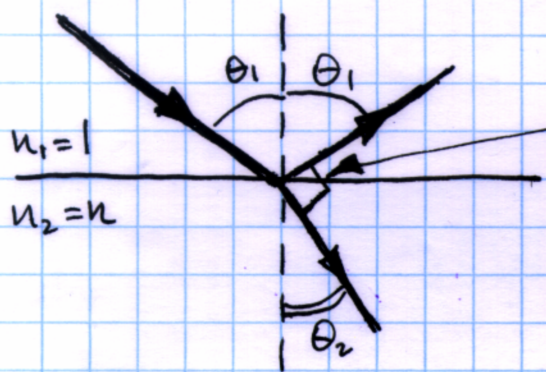
$\Rightarrow \quad \theta = \theta_c$!

(see the middle frame in the figure on the web)

Ex fiber optics



- polarization by reflection



a fact: reflected light is fully polarized when 90° between reflected and refracted beams:

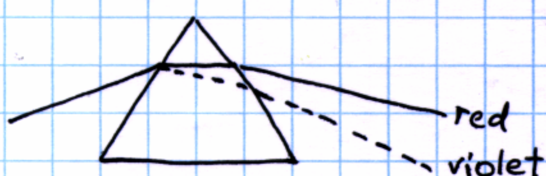
$$\theta_1 + 90^\circ + \theta_2 = 180^\circ$$

$$\Rightarrow \quad \theta_1 + \theta_2 = 90^\circ$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} = \frac{n}{1} = n \quad \Rightarrow \quad \frac{\sin\theta_1}{\underbrace{\sin(90-\theta_1)}_{=\cos\theta_1}} = n$$

$$\Rightarrow \quad \frac{\sin\theta_1}{\cos\theta_1} = \boxed{\tan\theta_1 = n} \quad \Rightarrow \quad \boxed{\theta_B = \arctan n}$$ Brewster's angle

- dispersion: n depends on wavelength (i.e. the speed of light in medium depends on λ)

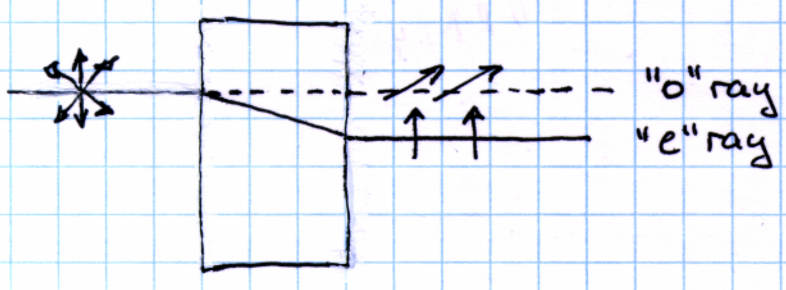


n_{glass} : 1.520 (red) ...... 1.538 (violet)

Ex a rainbow – see the Java applet

Note: the simplified picture of Fig. 26.17 assumes a single ray hitting each droplet, at a fixed distance from the center

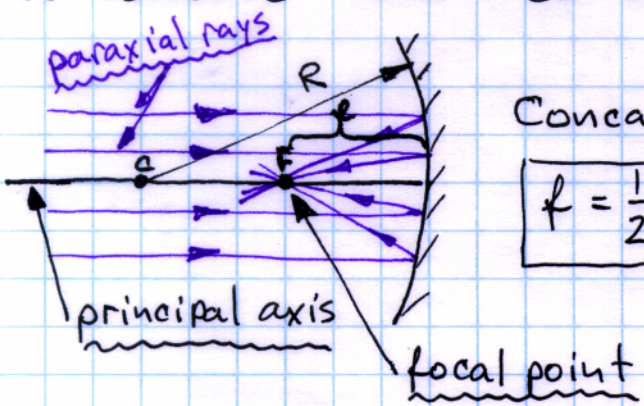
- birefringence: n depends on polarization



often, one of the two components is absorbed more than the other → dichroism

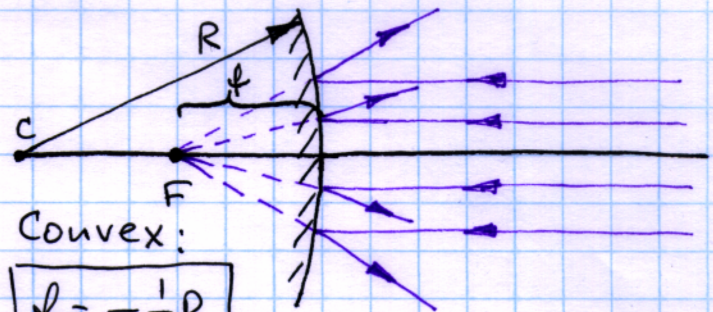
Spherical mirrors and lenses

- concave and convex spherical mirrors



Concave:

$$\boxed{f = \frac{1}{2}R}$$

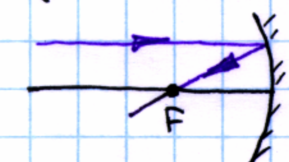


Convex:

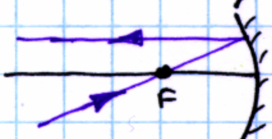
$$\boxed{f = -\frac{1}{2}R}$$

Note: only valid for paraxial rays; else need a parabolic not a spherical shape (spherical aberration)

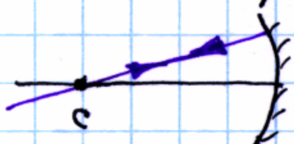
- image formation and ray tracing, in paraxial limit

1 

rays parallel to the principal axis converge at the focal point

2 

ray through the focal point is reflected parallel to the principal axis (reverse of 1)

3 

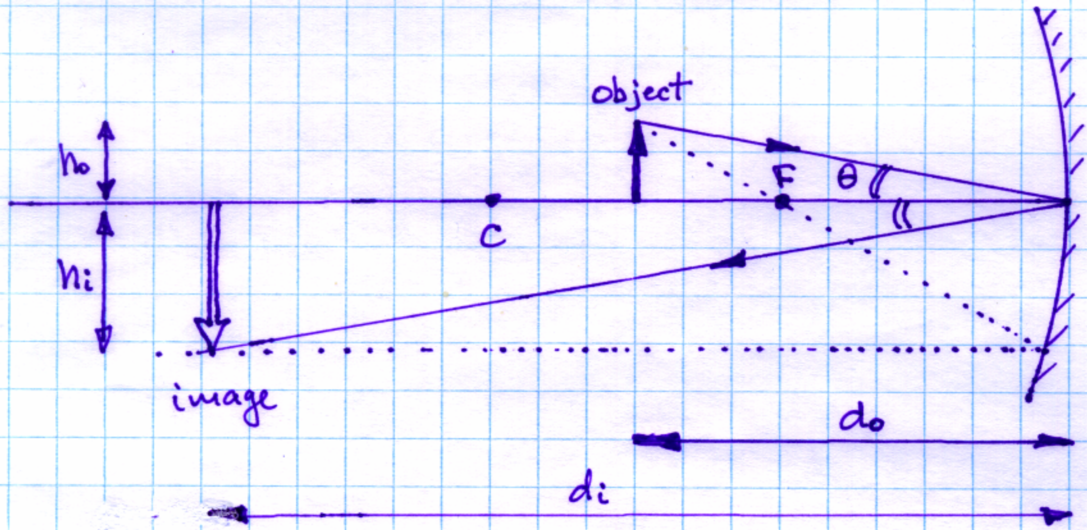
ray through the center of curvature reflects upon itself

similarly for convex mirrors

- real and virtual images:

$d_o > \emptyset$: real object, in front
$d_o < \emptyset$: virtual object, behind ← these arise in systems with more than one optical element (mirrors, lenses)
$d_i > \emptyset$: real image, in front
$d_i < \emptyset$: virtual image, behind

- magnification and mirror equation



From the drawing,

$$\tan\theta = \frac{h_o}{d_o} = \frac{-h_i}{d_i}$$

"–" sign, since image is inverted

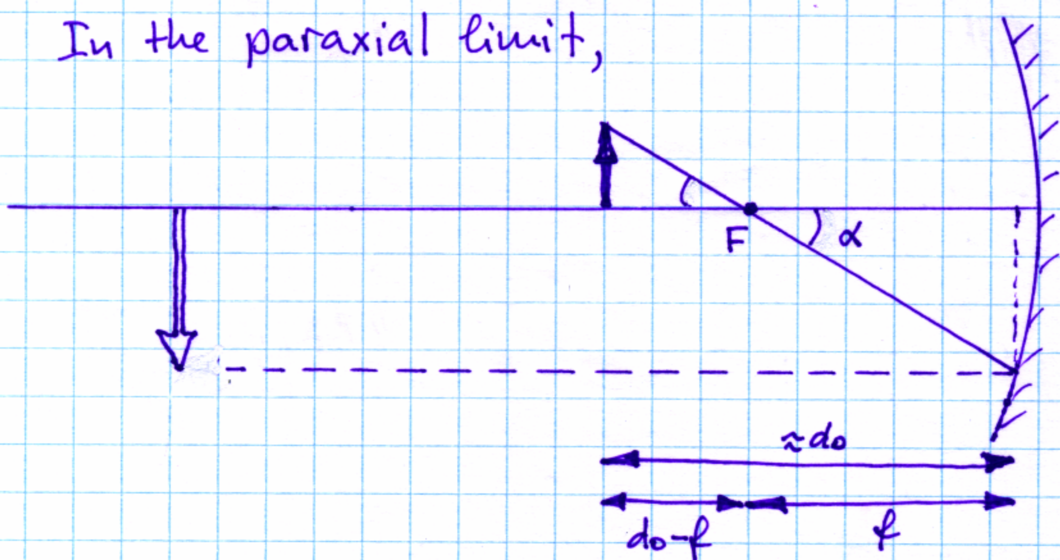
⇒ magnification

$$\boxed{M \equiv \frac{h_i}{h_o} = -\frac{d_i}{d_o}}$$

Convention : $M > 0$: image same orientation as object

$M < 0$: image inverted with respect to the object

In the paraxial limit,



$\tan\alpha =$

$= \dfrac{h_o}{d_o - f} =$

$= \dfrac{-h_i}{f}$

$$\Rightarrow \quad \frac{h_o}{-h_i} = \frac{d_o - f}{f} \quad \left.\right\} \quad \frac{d_o - f}{f} = \frac{d_o}{d_i}$$

Also: $M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

$$\Rightarrow \quad \frac{d_o}{f} - 1 = \frac{d_o}{d_i}$$

$$\Rightarrow \quad \frac{1}{f} - \frac{1}{d_o} = \frac{1}{d_i}$$

$$\Rightarrow \quad \boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = \frac{2}{R}}$$

works for both:
- concave $(f>0)$
- convex $(f<0)$

spherical mirror equation

<u>Ex</u> a concave mirror, $R = 30\,cm$
solve for object @ a) $d_o = 45\,cm$
b) $d_o = 20\,cm$ c) $d_o = 10\,cm$

a) <u>$d_o > R$</u>

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = \frac{2}{R} \qquad \text{all in cm}$$

$$\frac{1}{45} + \frac{1}{d_i} = \frac{2}{30} = \frac{1}{15} = \frac{3}{45}$$

$$\frac{1}{d_i} = \frac{2}{45}$$

$$d_i = \frac{45}{2} = 22.5\,cm > 0 \quad \text{real image}$$

$$M = -\frac{d_i}{d_o} = -\frac{1}{2} \quad \text{inverted, } \tfrac{1}{2}\text{-height}$$

b) $\underline{R > d_o > f}$: $\frac{1}{20} + \frac{1}{d_i} = \frac{1}{15}$

$$\frac{1}{d_i} = \frac{1}{15} - \frac{1}{20} = \frac{20-15}{20 \cdot 15} = \frac{\cancel{5}^{1}}{20 \cdot \cancel{15}_{3}} = \frac{1}{60}$$

$d_i = 60\,cm$ >0 real

$$M = -\frac{d_i}{d_o} = -\frac{60\,cm}{20\,cm} = -3.0$$

inverted, x3

c) $\underline{d_o < f}$: $\frac{1}{10} + \frac{1}{d_i} = \frac{1}{15}$

$$\frac{1}{d_i} = \frac{1}{15} - \frac{1}{10} = \frac{10-15}{15 \cdot 10} = \frac{-\cancel{5}^{1}}{\cancel{15}_{3} \cdot 10} = -\frac{1}{30}$$

$d_i = -30\,cm$ <0 virtual

$$M = -\frac{d_i}{d_o} = -\frac{-30}{10} = +3.0$$

upright, x3

other cases:

d) $d_o = 30\,cm = R$ $\Rightarrow$ $M = -1.0$, $d_i = d_o$

e) $d_o = 15\,cm = \frac{1}{2} R = f$ $\Rightarrow$ $d_i = +\infty$

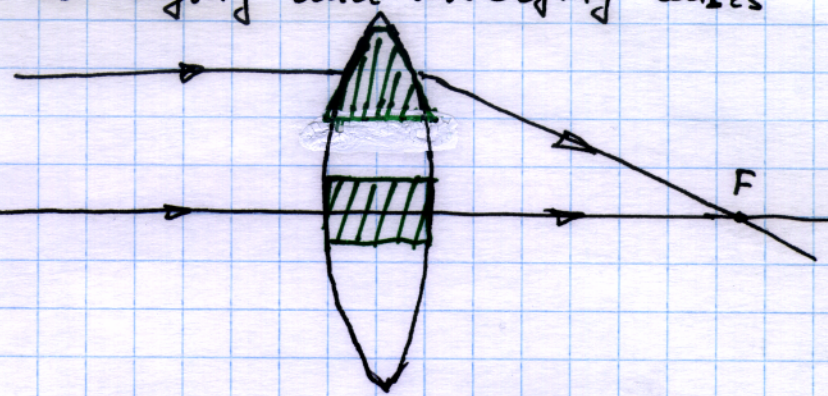
<u>Note</u>: plane mirror $\approx$ spherical mirror with a very large R !

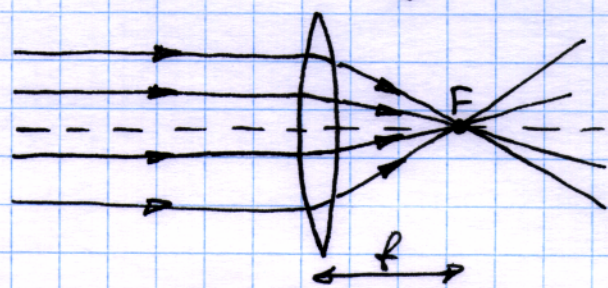
$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} = \frac{2}{R} \to \emptyset \text{ as } R \to \infty$$

$$\Rightarrow \frac{1}{d_o} = -\frac{1}{d_i}, \text{ or}$$

$$\boxed{d_i = -d_o}$$ as before

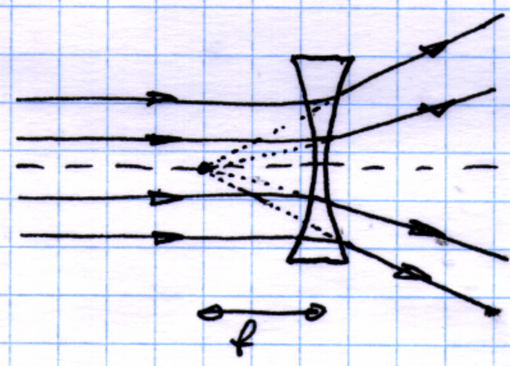
- converging and diverging lenses





Converging

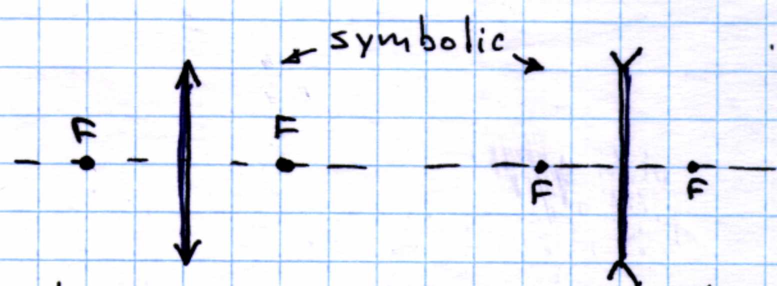
$f > 0$



Diverging

$f < 0$

Note : by symmetry, each lens has two focal points, one on the LHS, one on RHS



which one is used for ray tracing depends on the direction of the incoming ray.

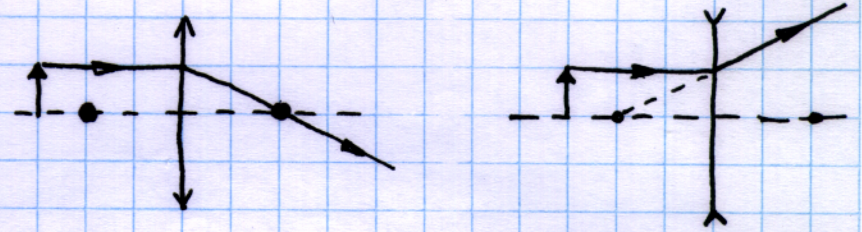
Convention :

$f > 0$: converging (not which side!)
$f < 0$: diverging.

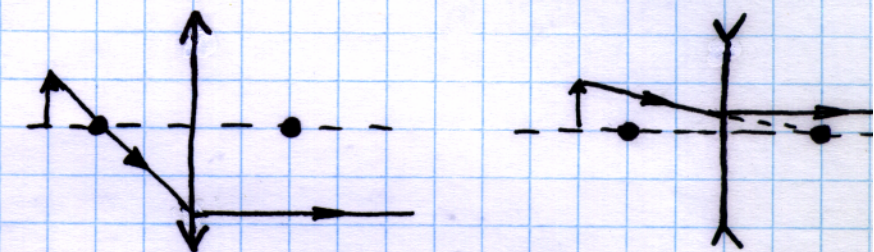
- image formation and raytracing

– special rays

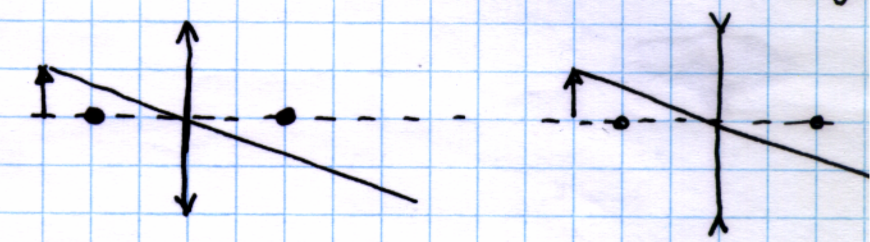
① ∥ principal axis → through focal pt.



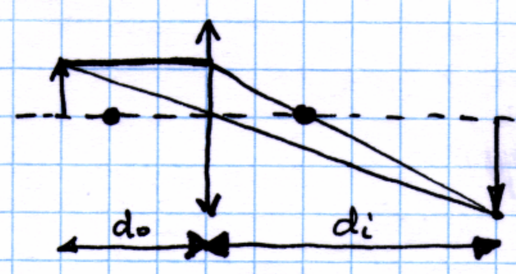
② through the focal pt → ∥ principal axis



③ through the center of the lens → stays straight

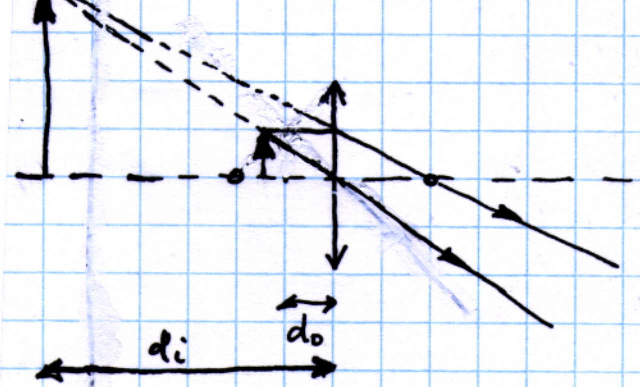


– real & virtual image



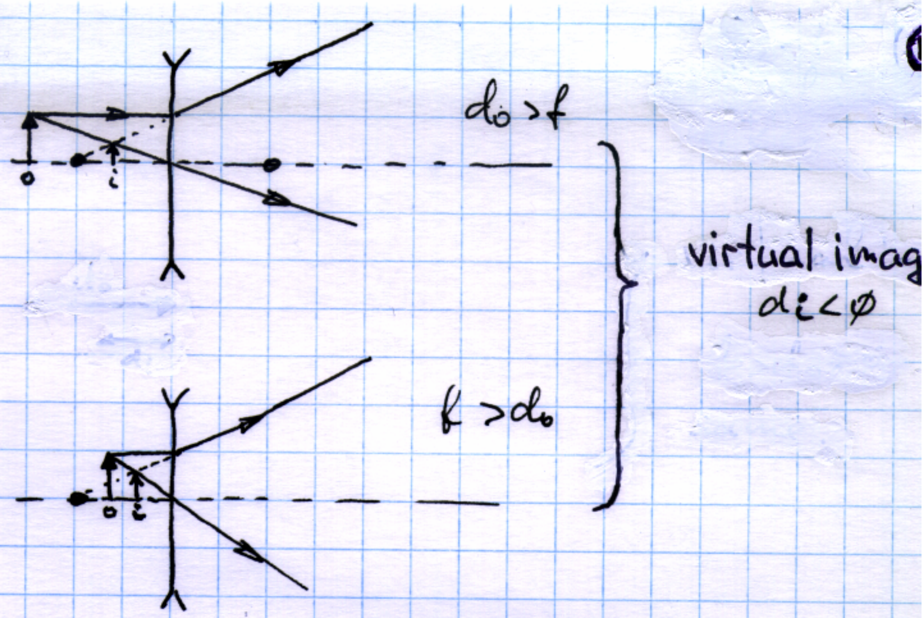
$d_o > f > 0$
real image
$d_i > 0$

Ex: projector



$f > d_o > 0$
virtual image
$d_i < 0$

Ex: magnifying glass



<u>Convention:</u>

$d_o > \phi$: real object, in front

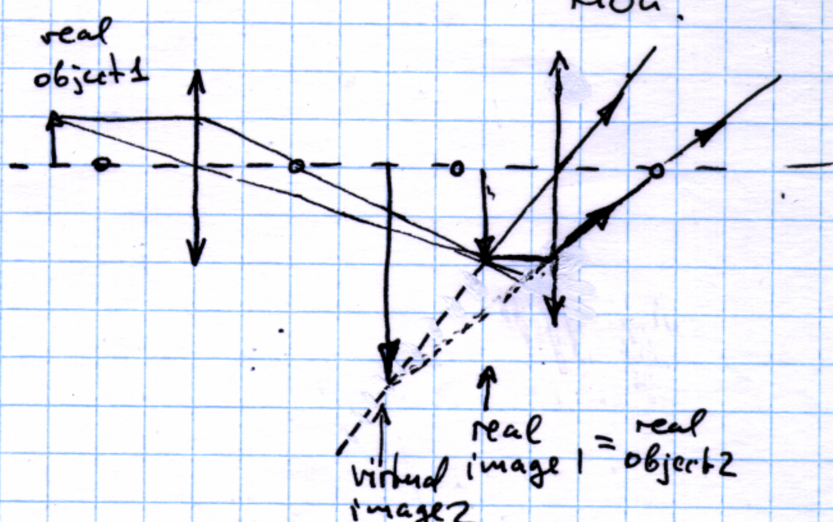
$d_o < 0$: virtual object, behind

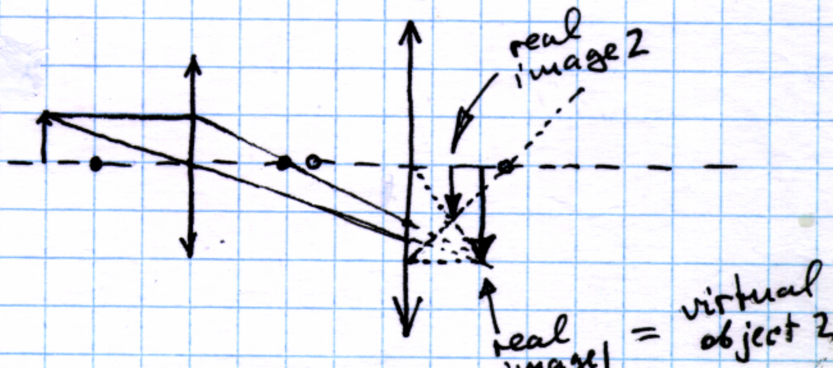
$d_i > \phi$: real image, on the other sid

of a real object

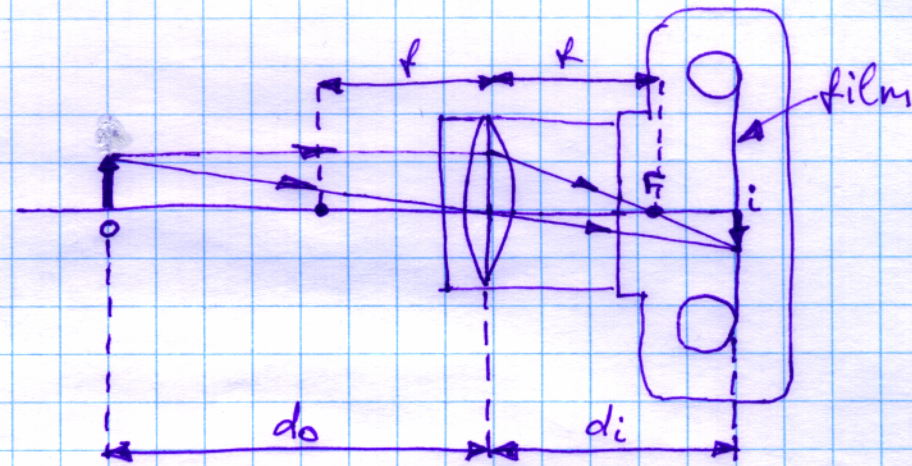
$d_i < \phi$: virtual image, on the same sid

- a virtual object: lenses in combination.





Ex macro lens on a camera



$$\boxed{\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}}$$

$f = 50.0\,mm$

$d_o = ?$

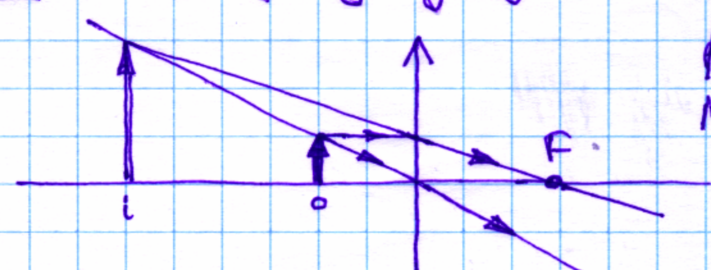
$d_i = 275\,mm$

$$\Rightarrow \underbrace{\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i}} = \frac{1}{50.0\,mm} - \frac{1}{275\,mm}$$

$$= \frac{5.5 - 1}{275\,mm}$$

$$\Rightarrow d_o = \frac{275\,mm}{4.5} = \underline{61.1\,mm}$$

Ex a magnifying glass



$f = 15\,cm$

$M = +3.0$

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = +3.0 \quad \Rightarrow d_i = -3.0 d_o$$

$$\Rightarrow \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \Rightarrow \frac{1}{d_o} - \frac{1}{3.0 d_o} = \frac{1}{f}$$

a) $\Rightarrow d_o = \frac{2}{3} f = \underline{10\,cm}$

b) $d_i = -3.0 d_o = \underline{-30\,cm}$ (virtual)