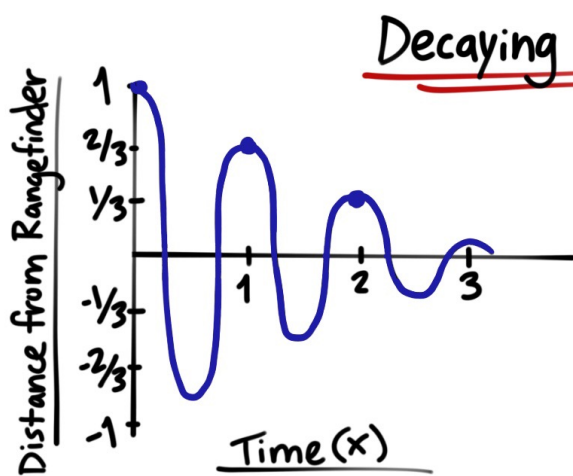# Tips for Fitting a Decaying Sinusoid

$y = A_0 e^{(-\gamma t)} cos(w_d t + \phi) \leftarrow$ Formula in lab manual
$y = A cos(Bx + C)e^{Dx} + E \leftarrow$ Formula in physicalab

When fitting a decaying sinusoid we need to account for the damping coefficient: D.

Decaying Sinusoid

$y = Ae^{-Dx}cos(Bx+C)+E$

The damping coefficient:

Unlike a regular cosine wave, a decaying cosine has an aditional parameter "D" known as the DAMPING COEFFICIENT

Estimating "D": If $D = \frac{1}{x}$, Ae^{-Dx} becomes $Ae^{\frac{-1}{x}(x)} \rightarrow Ae^{-1}$ or $\frac{A}{e}$
$\frac{A}{e} = \frac{A}{2.718...} \approx \frac{A}{3}$ So, when $D = \frac{1}{x}$ the amplitude is aprox. one third of its initial value.

In the case of this graph, the initial amplitude decreases by 2/3 after 2 sec. Therefore $D = \frac{1}{2}$

Solving the other parameters:

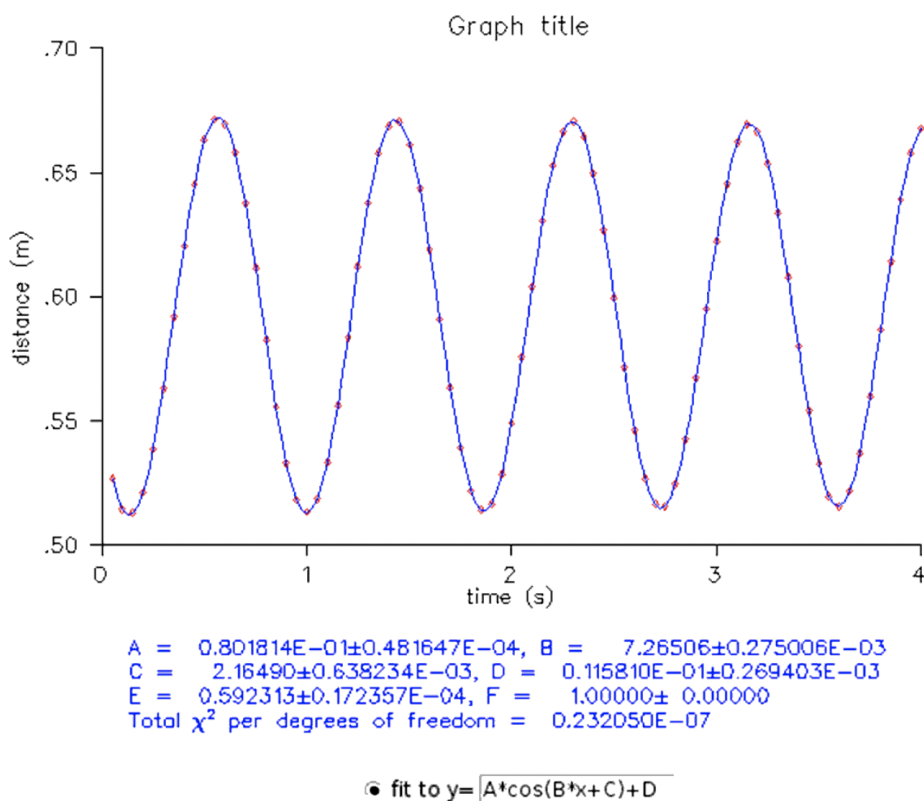1. In Physicalab your graph will look something like this:

Graph title
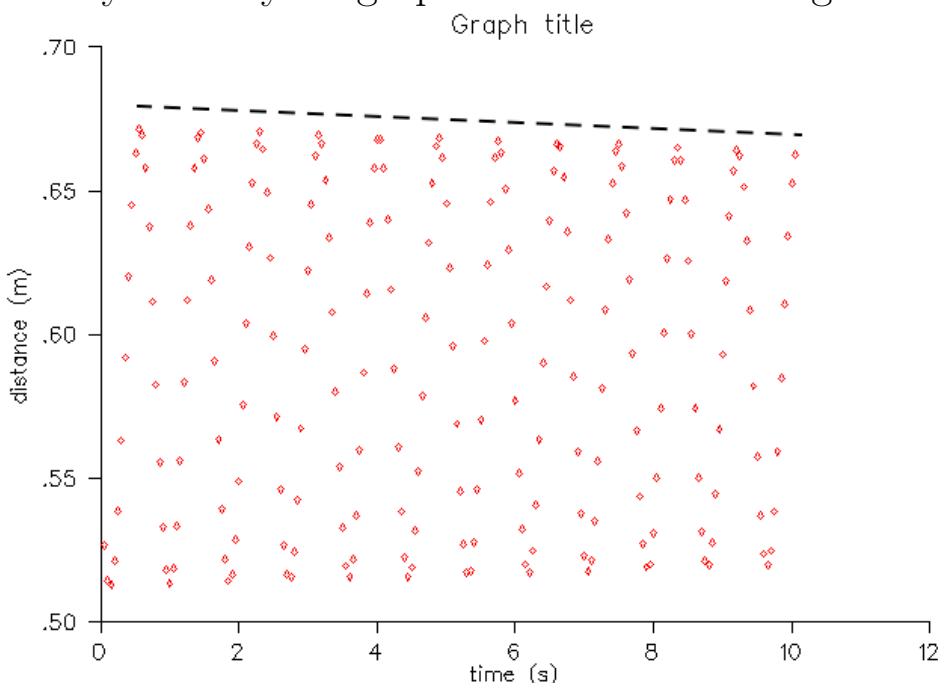
You can see the amplitude slowly decreases over time. However the first few cycles look very similar to an undamped sinusoidal function. Focus on these cycles to determine the period, initial amplitude and initial phase angle.

2. Constrain your graph and change it's scale to easily view the first few cycles.

Graph title
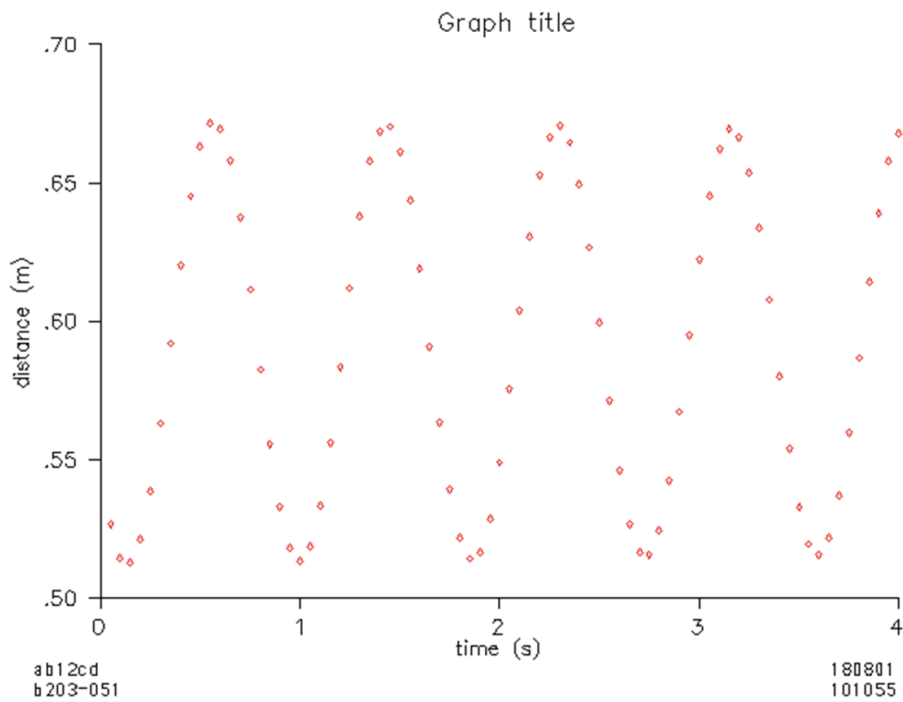
X= 0   to 4   in 4   steps ☐ autoscale   ✔ constrain to: (x<4)
Y= 0   to 0   in 0   steps ✔ autoscale

3. Make some initial guesses and try fitting a cosine wave to this data:

Graph title

A = 0.801814E−01±0.481647E−04, B = 7.26506±0.275006E−03
C = 2.16490±0.638234E−03, D = 0.115810E−01±0.269403E−03
E = 0.592313±0.172357E−04, F = 1.00000± 0.00000
Total χ² per degrees of freedom = 0.232050E−07

● fit to y= A*cos(B*x+C)+D

Now you can apply these known parameters to your original function and estimate your D value.

4. Select auto scale and remove any constraints.
   Select fit to $y = A\cos(Bx + C)e^{Dx} + E$. You have good estimates of your A, B, and C value, the last parameter you need to account for is D.

A =   0.800173E−01±0.572884E−04, B =   7.26530±0.129381E−03
C =   2.16442±0.736910E−03, D =   0.103204E−01±0.126756E−03
E =   0.592175±0.199570E−04, F =   1.00000± 0.00000
Total χ^2 per degrees of freedom =   0.798960E−07

⦿ fit to y= A*cos(B*x+C)*exp(-D*x)+E   ▾ ⬚ constrain to: (x<4)

initial guesses: A= 0.8   B= 7.3   C= 2.16   D= 0.1   E= 1   F= 1

X= 0   to 4   in 4   steps ☑ autoscale ⬚ X grid ⬚ X log ☑ X label time (s)
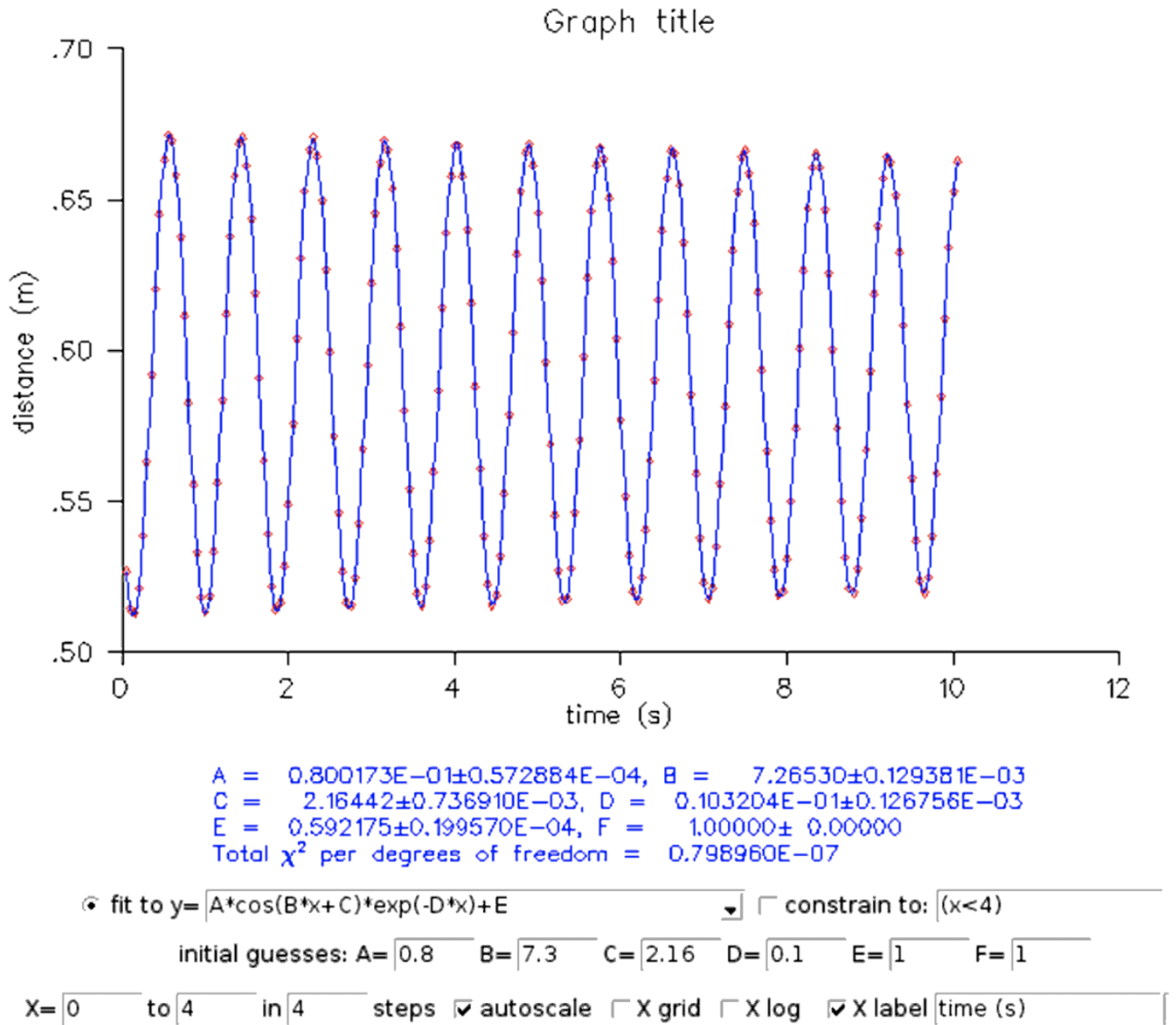
You may need to continue to adjust your estimations.