Chapter 21: Electric potential

Acknowledgements: Several Images and excerpts are taken from
College Physics: A strategic approach, *Pearson Education Inc*

Electric Potential Energy

A charged particle acquires **electric potential energy** when it is brought near other charges.

Lightning is a dramatic example of conversion of electric potential energy to light and thermal energy.

Looking Back ◀◀
10.2–10.4 Work, kinetic energy, and potential energy

Electric Potential

The **electric potential** at a given location determines the electric potential energy that a charge would have if placed there. Electric potential differences are caused by the *separation of charge.*

Batteries create a charge separation by chemical means, leading to a potential difference between their terminals.

Using the Electric Potential

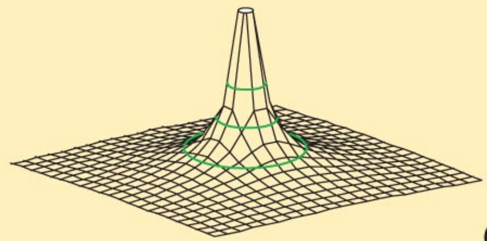
Depending on the sign of its charge, a charged particle speeds up or slows down as it moves through a potential difference.

X rays are produced when electrons, accelerated through a large potential difference, collide with a metal target.

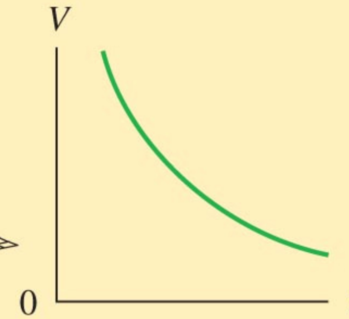
Looking Back ◀◀
20.3 Conservation of energy

Calculating the Electric Potential

You'll learn to calculate the electric potential for several important charge distributions.



Elevation graph

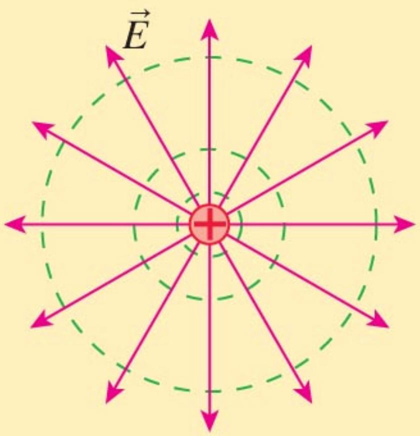


Potential graph

Because the electric potential is an abstract idea, we'll develop several different ways of *visualizing* the electric potential, as shown here for the important case of the potential due to a point charge.

Connecting Potential and Field

The electric field and electric potential are intimately connected. You'll learn how to move from the field representation to the potential representation and back again.



The electric field and the electric potential can be related to each other graphically.

Looking Back ◀◀
20.4 The electric field

Capacitors

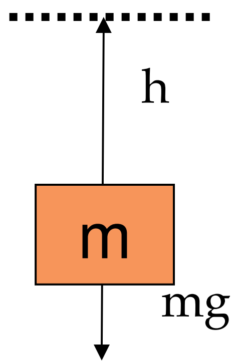
Capacitors store charge and electric potential energy. They're used in devices ranging from computers to defibrillators.

Each cylindrical element in this circuit is a capacitor, capable of rapidly storing and releasing charge and energy.

Looking Back ◀◀
20.5 Parallel-plate capacitors

What is Potential Energy?

Think! a object of mass m is lifted against the gravitational force – *it will have a potential energy*



To lift this object at height h, we need to work against the gravitational force (= mg),

Where g is gravitational constant = 9.8 m/s^2

At height h, it gains the extra energy, which is called potential energy

Potential Energy change = work done

= Force x displacement

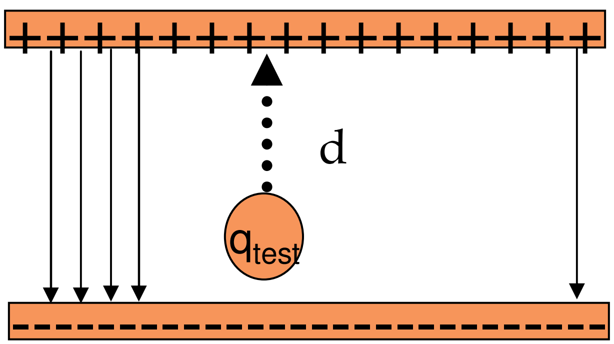
= mg x h

What about charged object in E-fields?

What is Electric Potential Energy (U_{elec})?

Just like gravitational force, electrostatic force is conservative

To see how the potential energy of of a charge changes with position, think that we placed a positive charge q_{test}, in uniform electric field created by a parallel plate capacitor



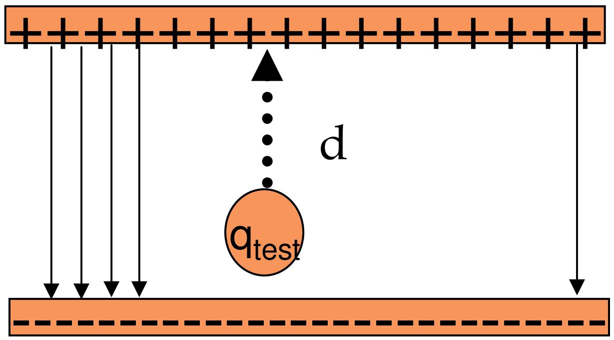
- It will accelerate towards negative plate
- To bring the q_{test} upwards, we need to apply external force (F_{ext})
- How much? Equivalent to the electrostatic force exerted on the q_{test} due to the electric field E

$F_{electrostatic} = E\, q_{test}$

- Work done = Force x displacement

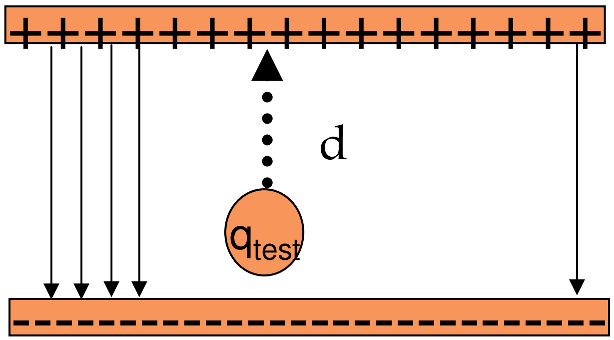
This work done by external force = the increase in electric potential energy (U_{elec}) of $q_{test} = Eq_{test}d$

Electric Potential Energy continued…



- Gravitational potential energy always increases, when you lift an object
- Electrostatic potential energy of charge particle depends on the direction of electric field
- Electric potential energy of a charge particle will increase when we move it against the direction that the electrostatic force would normally tend to move it

What is Electric Potential (V)?



1. *Electric potential depends on the source charges that create E-field and not on the test charge*
2. *It is a scalar*
3. *It tells how much a charge will gain energy when it moves from a point 1 and 2*

- Electric potential, also called voltage, is related to U_{elec} in the same way, as electric field E is related to the electrostatic force
- The change in electric potential (ΔV) is defined as

$$\Delta V = \frac{U_{elec}}{q_{test}}$$

Units of electric potential = J/C (also called volt, V)

1 J/C = 1 V

Example Problem

A uniform electric field of 1000 N/C is established between a parallel plate capacitor. A particle with charge of +0.005C is moved 3 cm from the negatively charged plate to the top plate.

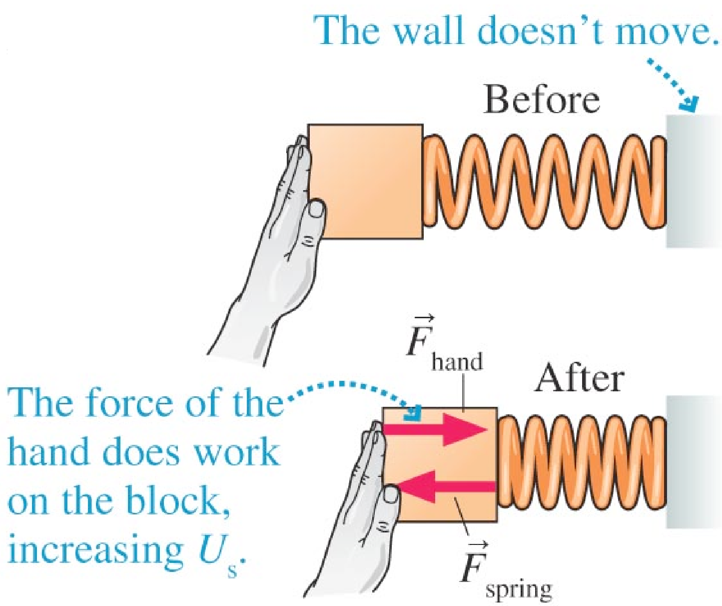
1. Change in electric potential energy U_{elec}?
$U_{elec} = Eq_{test}d$
[Given q_{test} = 0.005C; d= 3cm= 0.03m, E= 1000N/C]
U_{elec}= (1000N/C)(0.005C)(0.03m) = 0.15J

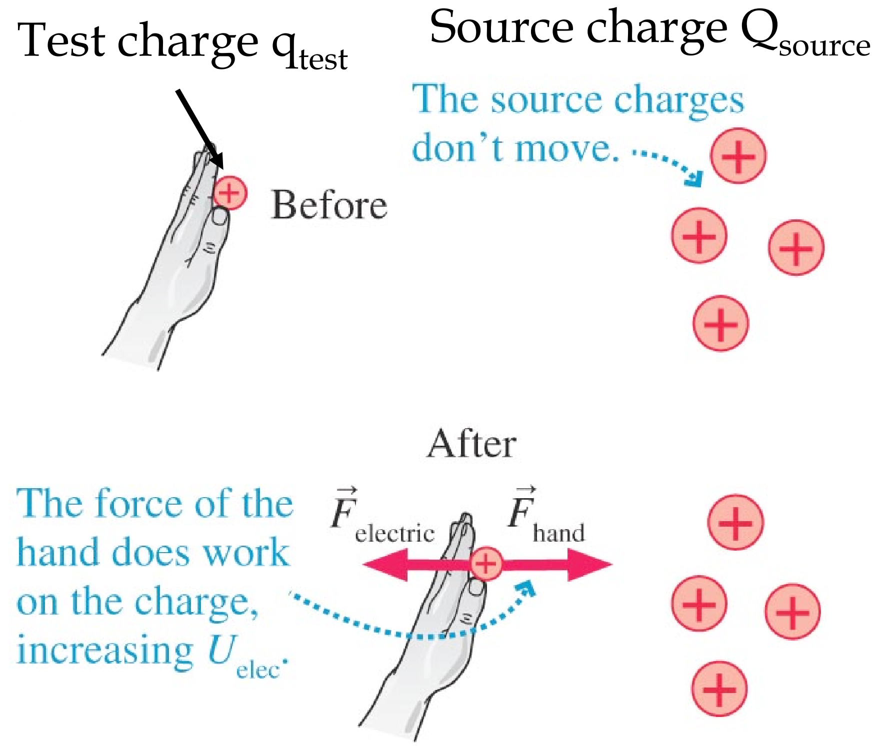
2. Change in electric potential (ΔV) = U_{elec}/q_{test}
= 0.15J/0.005C = 30V

The electric field tells us what the force *would be* if a charge were placed at this point.

Is there a quantity associated with each point around the source charges that would tell us the electric potential energy of a charge placed at that point?

U_{elec} associated with point charges

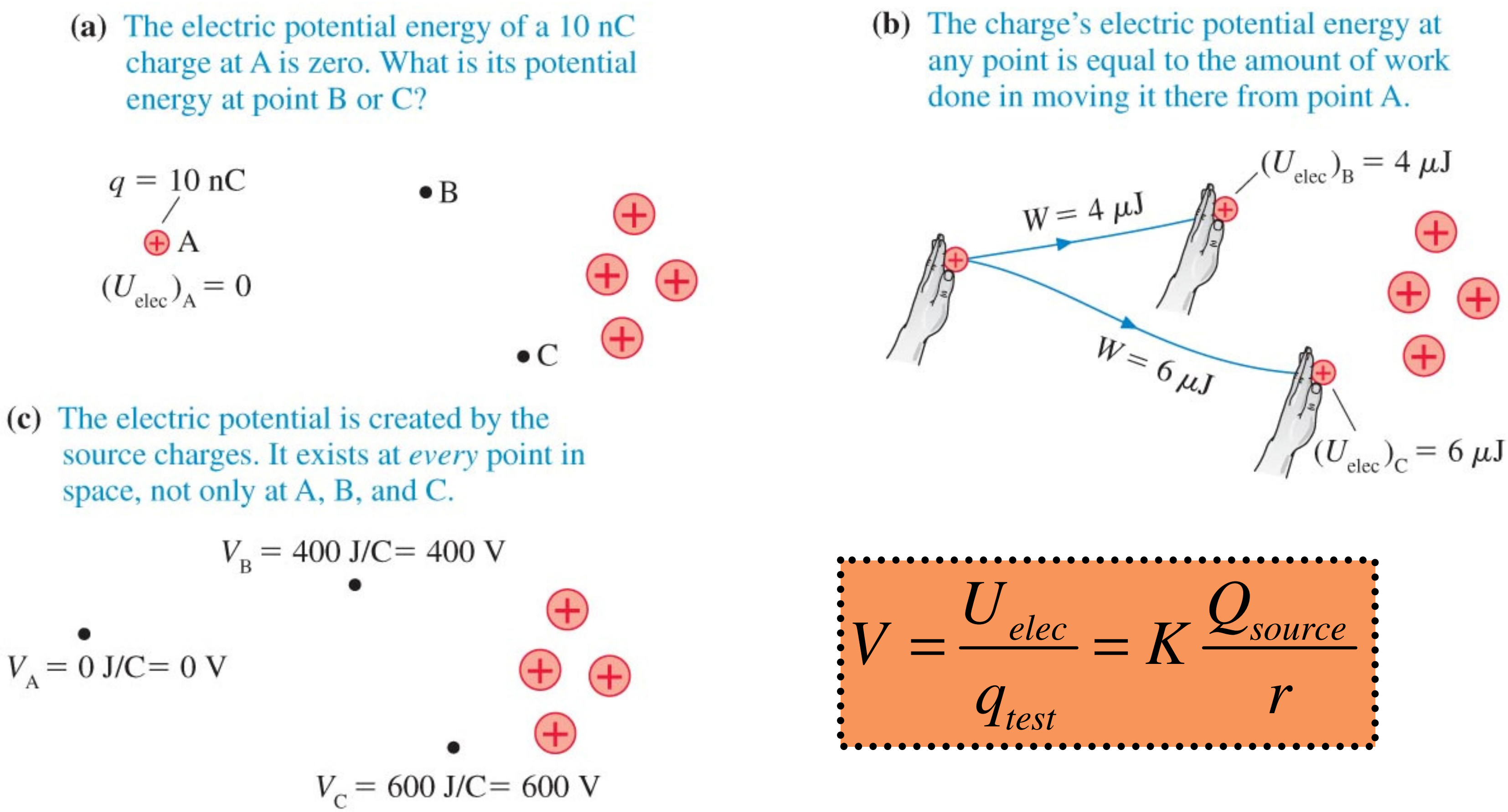




$$U_{elec} = K\frac{q_{test}Q_{source}}{r}$$

U_{elec} energy needed to bring q_{test} & $Q_{sources}$ together from infinity

Electric Potential continued…



$$V = \frac{U_{elec}}{q_{test}} = K\frac{Q_{source}}{r}$$

Electric potential and potential energy due to multiple point charges

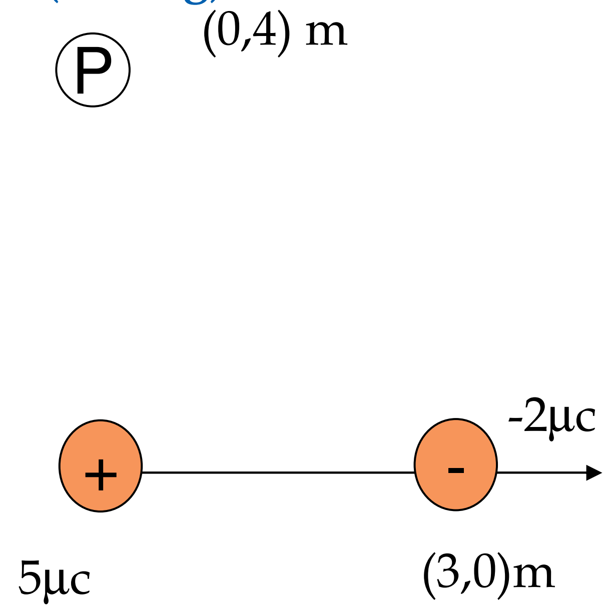
Multiple charges:

1. Apply superposition rule
2. Calculate potential energy for every pair of the charges
3. Sum algebraically

$$U_{elec} = K \sum \frac{q_i q_j}{r_{ij}}$$

$$V = K \sum \frac{q_i}{r_i}$$

Problem: Two charges q1 and q2 lie in x-axis (see Fig)



(a) What is total electric potential at P

$$V_P = K\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

Vp= $(9\times10^6 Nm^2C^{-2})[5\times10^{-6}C/4m - 2\times10^{-6}C/5m]$

$= 7.65 \times 10^3$ V

(b) How much work will be required to bring a 3rd charge at point P?

$= q_3V_p$

Say $q_3 = 3\mu C$; Work = 2.3×10^{-2} J

TABLE 21.2 Distinguishing electric potential and potential energy

The *electric potential* is a property of the source charges. The electric potential is present whether or not a charged particle is there to experience it. Potential is measured in J/C, or V.

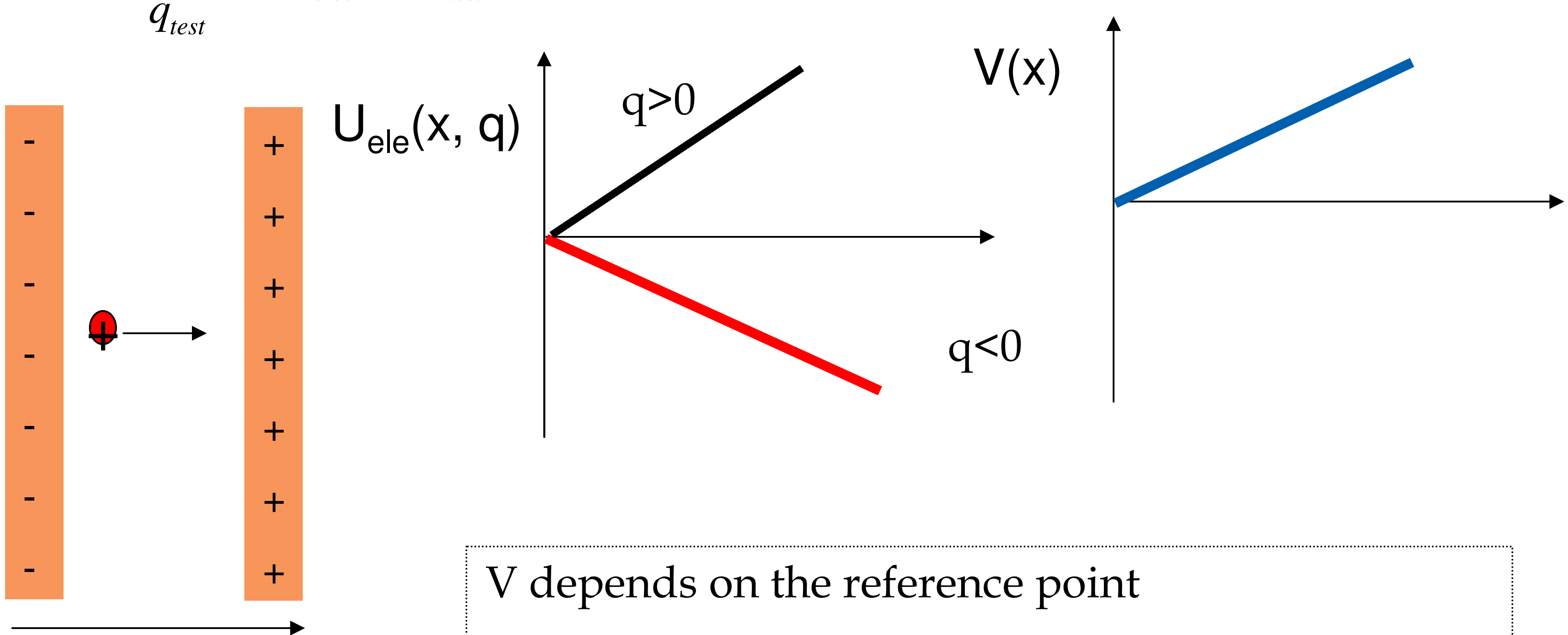
The *electric potential energy* is the interaction energy of a charged particle with the source charges. Potential energy is measured in J.

TABLE 21.1 Typical electric potentials

Source of potential	Approximate potential
Cells in human body	100 mV
Battery	1–10 V
Household electricity	100 V
Static electricity	10 kV
Transmission lines	500 kV

Potential of a Parallel-Plate Capacitor

$$V = \frac{U_{elec}}{q_{test}} \Rightarrow U_{elec} = Vq_{test}$$



V depends on the reference point

U_{elec} also depends on the sign of the test charge

Find electric potential (V) of the capacitor at point x?

$$U_{elec} = Work\,done = qEx$$

$$E_{capacitor} = \frac{Q}{\varepsilon_o A}$$

$$\boxed{\Rightarrow V = \frac{Q}{\varepsilon_o A}x} \qquad \text{--------(1)}$$

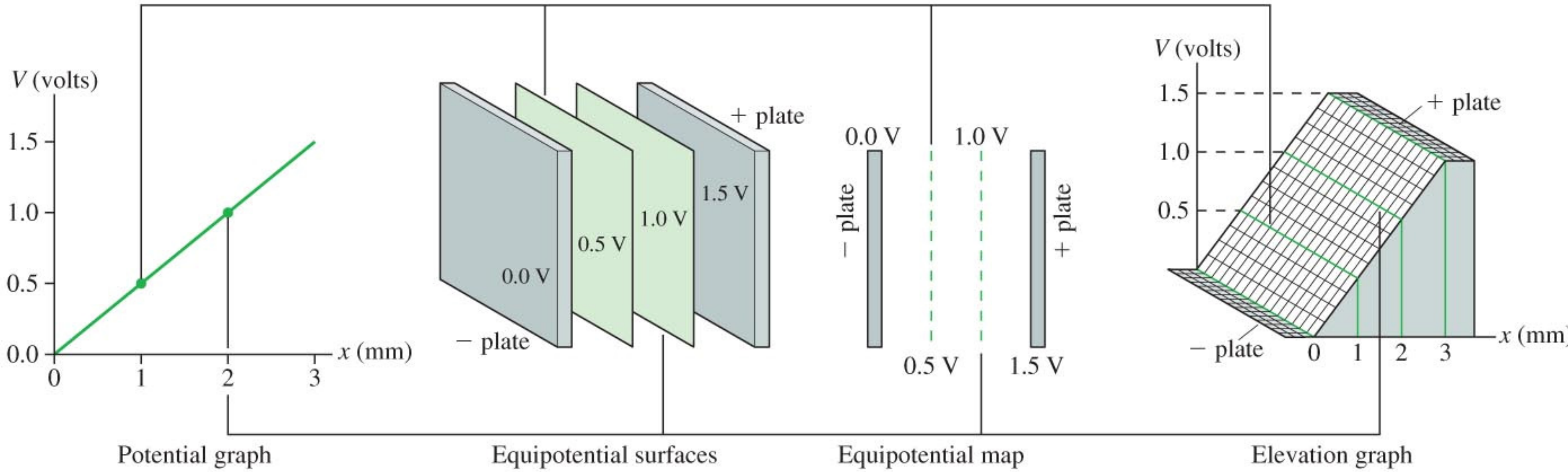
Potential difference (ΔV) between two plates separated by d ?

$$\Delta V_c = V_+ - V_- = Ed$$

$$\Rightarrow E_{capacitor} = \frac{\Delta V_c}{d} \qquad \text{--------(2)}$$

$$\boxed{V = \frac{x}{d}\Delta V_c} \qquad \text{--------(3)}$$

The Potential Inside a Parallel-Plate Capacitor



$$V = \frac{U_{\text{elec}}}{q} = Ex = \frac{Q}{\epsilon_0 A} x$$

Application

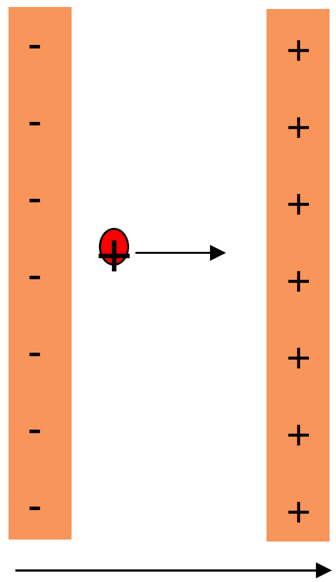
Energy gained by a charged particle going through a potential difference

Kinetic energy (K.E.) of a particle of mass m and velocity v = ½ mv^2

Find initial kinetic energy = ½ mv_i^2

Final kinetic energy = ½ mv_f^2

Potential energy change = $q(V_f - V_i)$



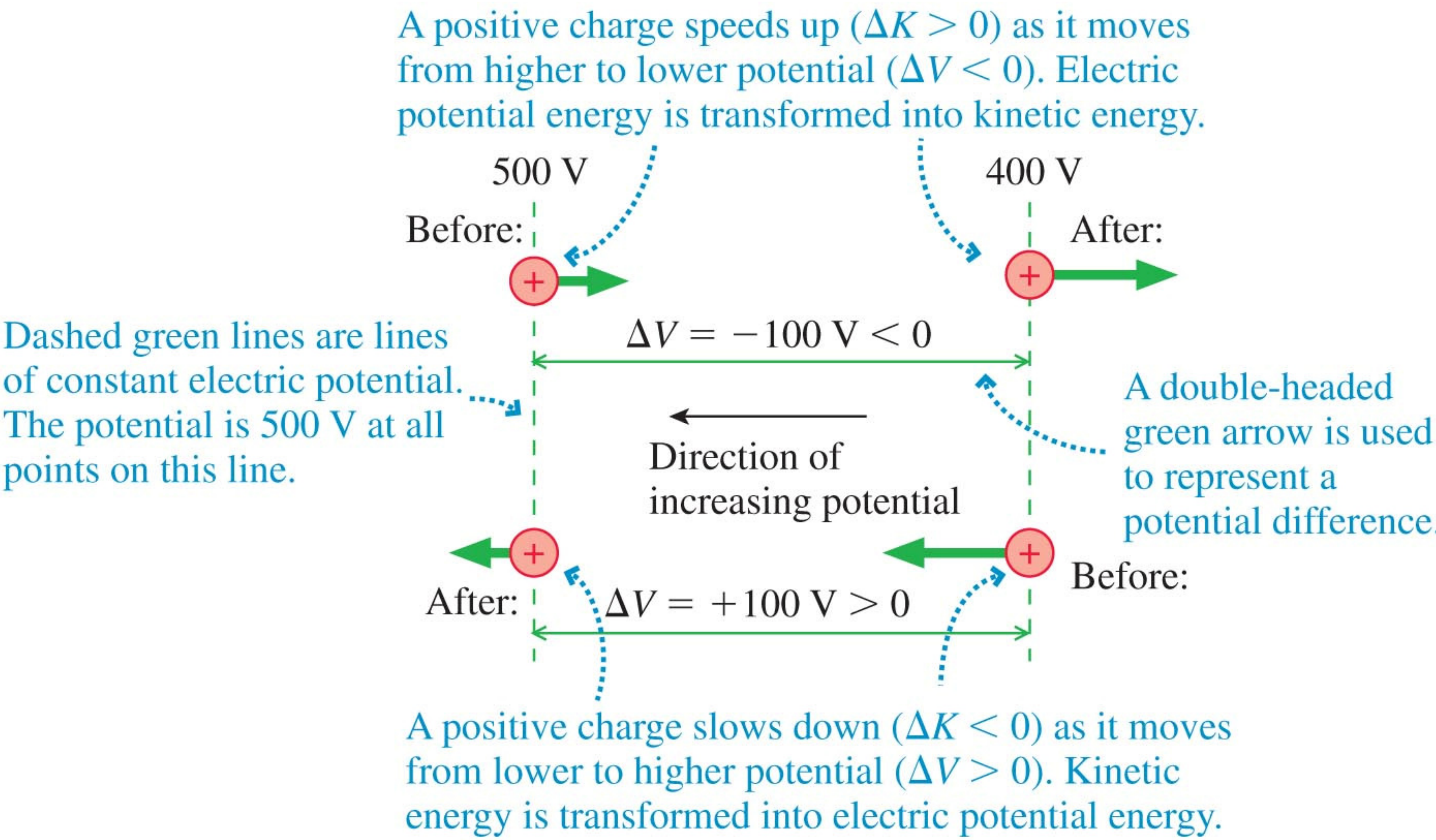
Since energy is conserved

Initial K.E. + initial Potential Energy

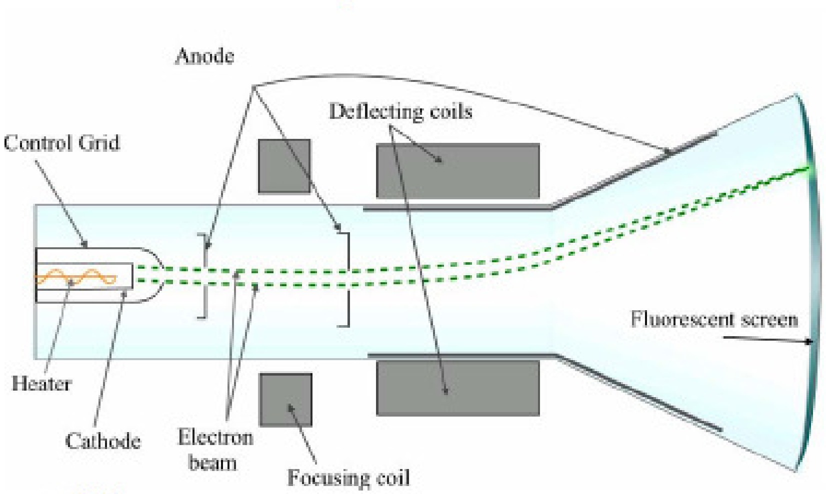
= Final K.E. + Potential energy

$\frac{1}{2} mv_i^2 + qV_i = \frac{1}{2} mv_f^2 + qV_f$

Charged Particle Moving Through a Potential Difference



Cathode Ray Tube



ΔV

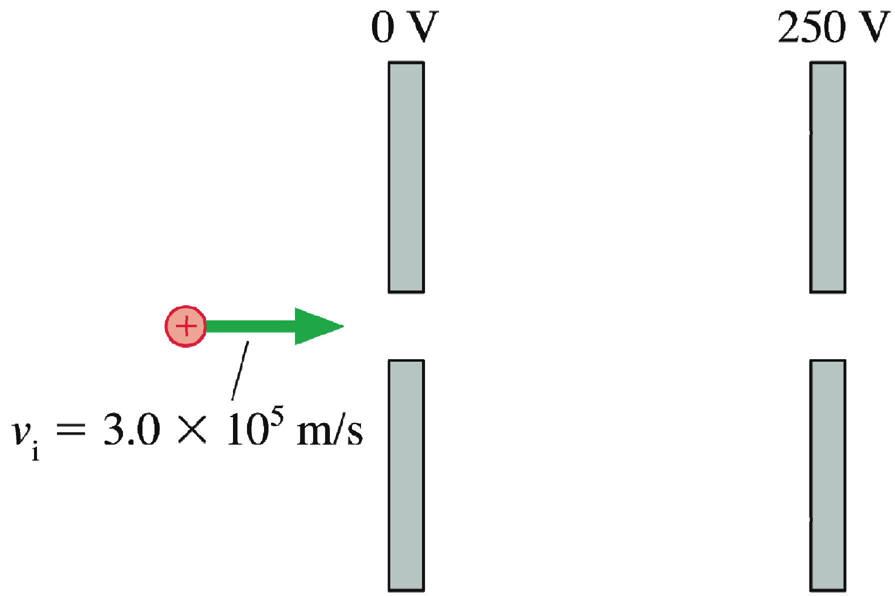
Brookhaven National lab
Relativistic Heavy Ion Collider

$E_{kin} \sim 10$ keV (10^4eV)

$E_{kin} \sim 100$ GeV (10^{11}eV)

Problem

A parallel-plate capacitor is held at a potential difference of 250 V. A proton is fired toward a small hole in the negative plate with a speed of 3.0×10^5 m/s. What is its speed when it emerges through the hole in the positive plate? (Hint: The electric potential outside of a parallel-plate capacitor is zero).



PROBLEM-SOLVING STRATEGY 21.1

Conservation of energy in charge interactions

PREPARE Draw a before-and-after visual overview. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

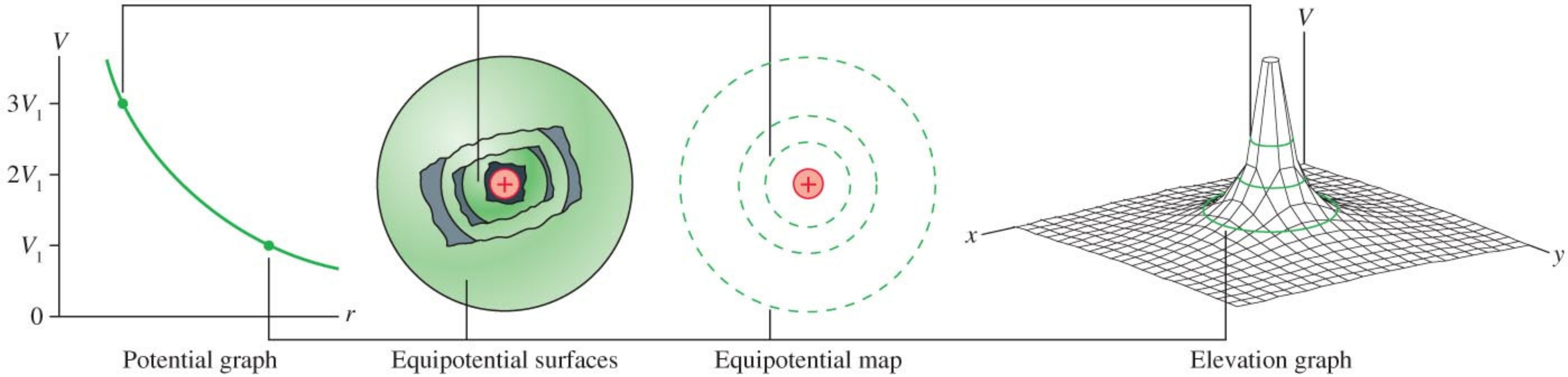
SOLVE The mathematical representation is based on the law of conservation of mechanical energy:

$$K_f + qV_f = K_i + qV_i$$

- Find the electric potential at both the initial and final positions. You may need to calculate it from a known expression for the potential, such as that of a point charge.
- K_i and K_f are the total kinetic energies of all moving particles.
- Some problems may need additional conservation laws, such as conservation of charge or conservation of momentum.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Electric Potential of a Point Charge



$$V = K\frac{q}{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$

Electric potential at distance *r* from a point charge *q*

Electric Potential: Charged Sphere

Outside of a sphere of charge Q the potential has the same form as for a point charge Q:

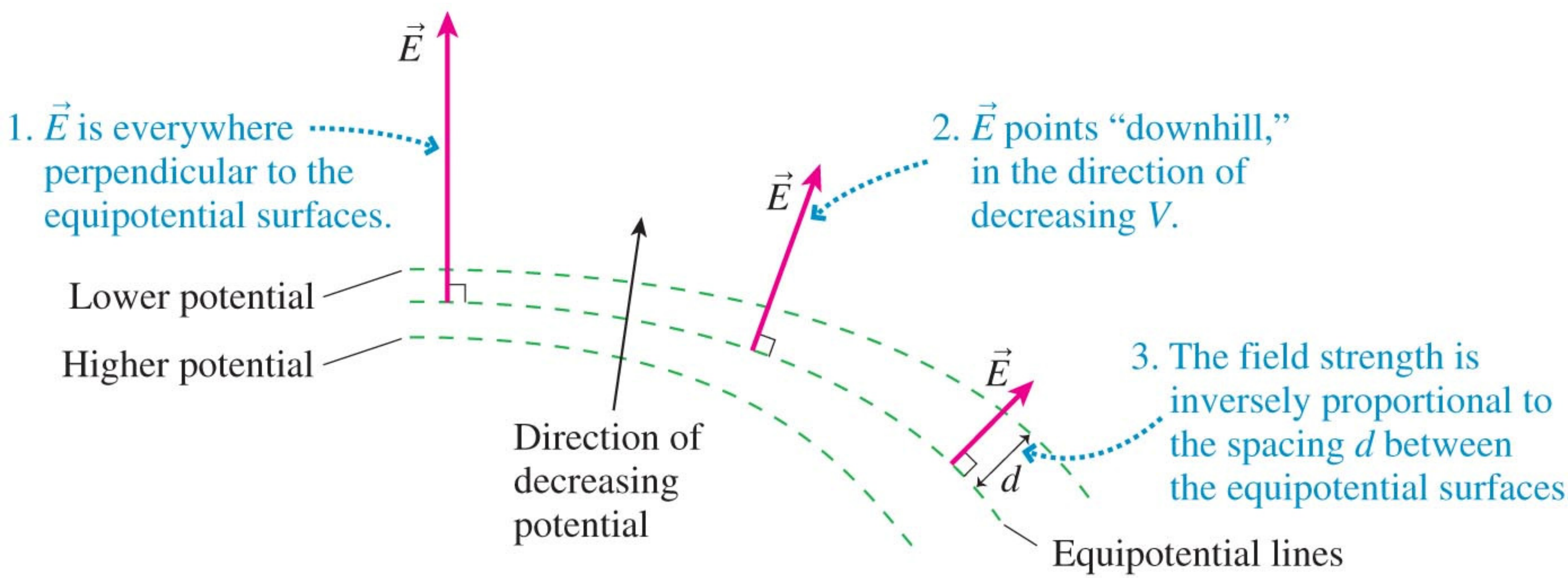
$$V = K\frac{Q}{r} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

Electric potential at a distance $r > R$ from the center of a sphere of radius R and with charge Q

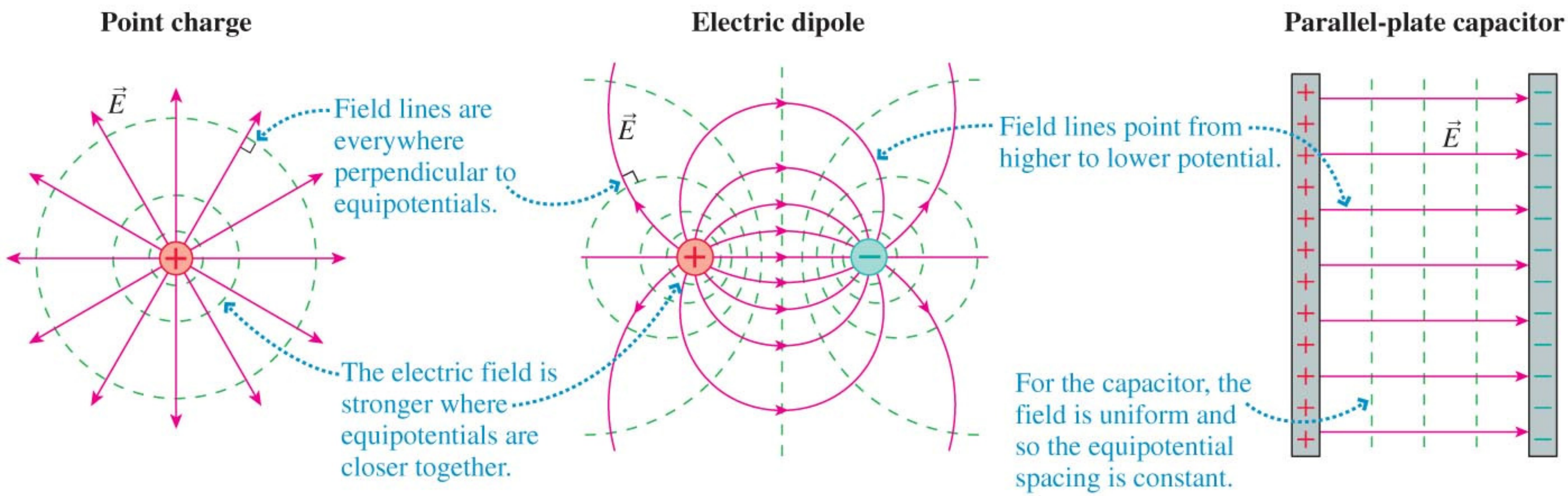
If the sphere has radius R and the potential at its surface is V_0, then the potential a distance r from its center can also be written

$$V = \frac{R}{r}V_0$$

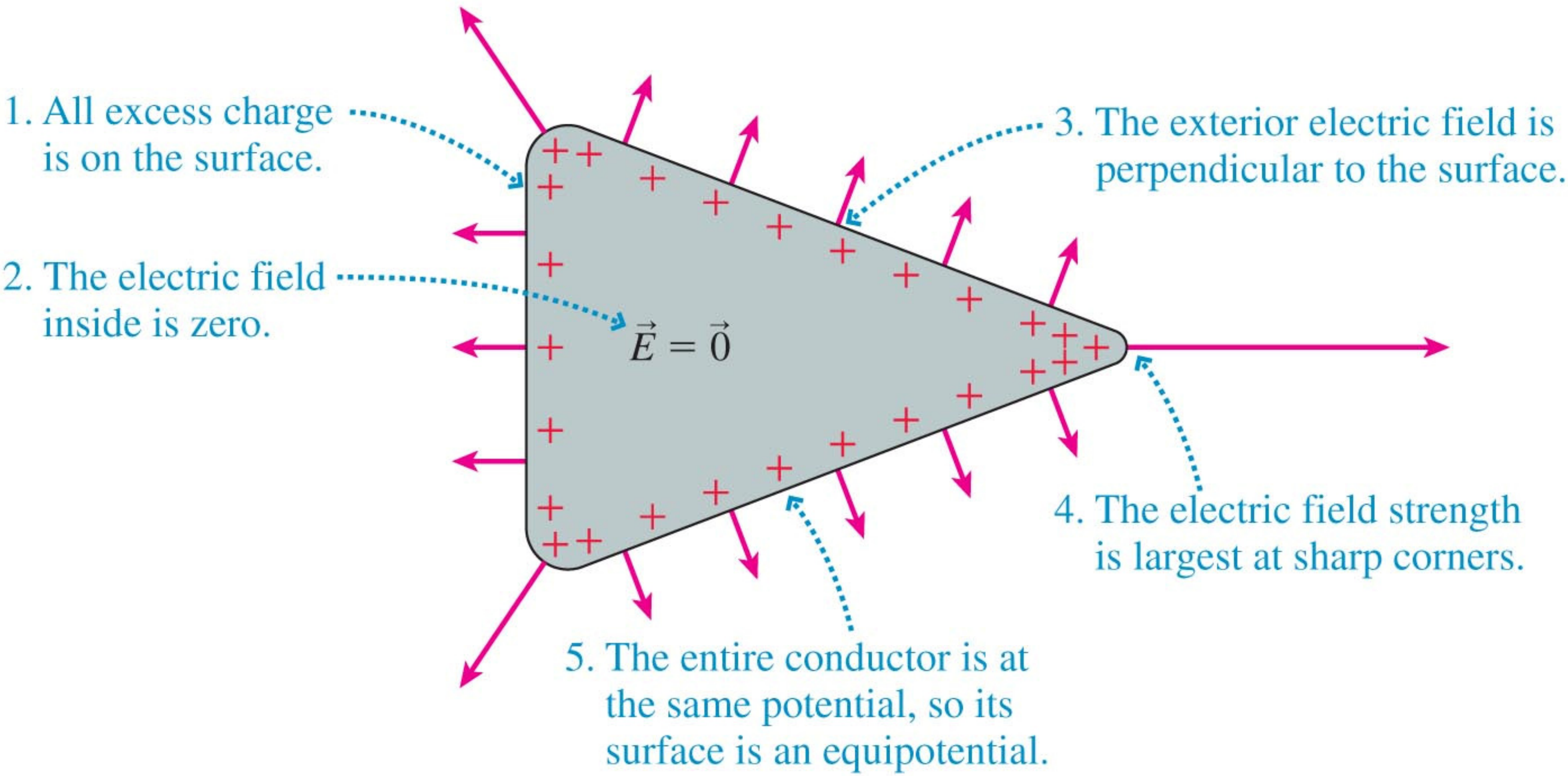
Relationship between Electric Potential and Electric Field



Electric Potential and Electric Field for Three Important Cases



A Conductor in Electrostatic Equilibrium



A Topographic Map

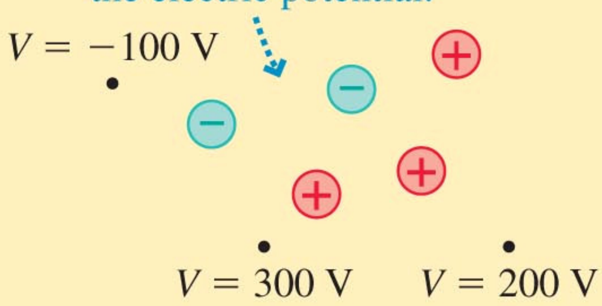
Chief Mountain in Glacier National Park, Montana

2200 ft
2100 ft

Summary

Electric Potential and Potential Energy

The electric potential V is created by charges and exists at every point surrounding those charges.



When a charge q is brought near these charges, it acquires an electric potential energy

$$U_{\text{elec}} = qV$$

at a point where the other charges have created an electric potential V.

Energy is conserved for a charged particle in an electric potential:

$$K_f + qV_f = K_i + qV_i$$

or

$$\Delta K = -q\Delta V$$

Summary

Sources of Potential

Potential differences ΔV are created by a *separation of charge*. Two important sources of potential difference are

- A *battery,* which uses chemical means to separate charge and produce a potential difference.
- The opposite charges on the plates of a *capacitor,* which create a potential difference between the plates.

The electric potential of a point charge q is $V = K\dfrac{q}{r}$

Connecting potential and field