Assignment No. 1

Physics 2P20

Due September 27, 2023, 09:30, hardcopy in class

1. *EFTS*: Show that
$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y)\hat{\imath} + (A_z B_x - A_x B_z)\hat{\jmath} + (A_x B_y - A_y B_x)\hat{k}$$
using the properties of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

2. *EFTS*: Show that
$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$
using the properties of the unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$.

3. Find a unit vector perpendicular to $\vec{A} = (\hat{\imath} + \hat{\jmath} - \hat{k}) = (1, 1, -1)$ and $\vec{B} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) = (2, -1, 3)$. What is its magnitude?

4. Find the area of the triangle with vertices $(1, -1, 0)$, $(2, 1, -1)$, and $(-1, 1, 2)$.

5. A particle moves along the curve $y = Ax^2$ so that its position is given by $x = Bt$.
   (a) Find the position vector of the particle in the form
   $$\vec{r}(t) = x(t)\,\hat{\imath} + y(t)\,\hat{\jmath}$$
   (b) Calculate the speed $v = |\vec{v}|$ of the particle along this path at an arbitrary instant t.

6. A particle moves outward along a spiral. Its trajectory is given by $r = A\theta$, where A is a constant, $A = (1/\pi)$ m/rad. θ increases in time according to $\theta = \alpha t^2/2$, where α is a constant.
   (a) Sketch the motion, and indicate the approximate velocity and acceleration at a few points.
   (b) Show that the radial acceleration is zero when $\theta = 1/\sqrt{2}$ rad.
   (c) At what angles do the radial and tangential accelerations have equal magnitude?

7. Make a rough sketch of the following functions, specified in polar coordinates:

   (a) $r = \sin\theta$

   (b) $r = 2a/\sin 2\theta$

   (c) $r = a(1 + \cos\theta)$

   (d) $r = \sin(a\theta^2)$

   where a is a positive constant.

   Try to use several (very few!) special points, and pay attention to the limiting behaviour of the function. It helps to sketch the Cartesian plot of r *vs.* θ first. Explain your reasoning as required.

   Use `extrema` (or another graphics package, if you prefer, such as `maple`, `gnuplot`, `MATLAB/octave`, *etc.*) to confirm the validity of your sketches. Try several "interesting" values of a. Make sure you are using enough points to define your functions in the regions where they change rapidly.

   The following should refresh your memory on a few of `extrema` commands. For more information, consult the built-in Help and/or the notes from the introductory labs.

```
define\constants
theta=[0:Pi:0.01]
r=cos(theta)
graph theta,r
pause

x=r*cos(theta)
y=r*sin(theta)
set aspectratio 1
scales -1,1,4,-1,1,4
graph x,y
set curvelinetype 9
zerolines
```