Assignment No. 2

Physics 2P20

Due October 18, 2023, 09:30

1. *EFTS*: Ex.5, the two-pulley system (K&K2 Pr. 2.8). Complete the solution started in class to show that

$$\ddot{x}_m = \frac{2M - m}{4M + m}\, g\,.$$

2. A particle of mass m moves along a frictionless, horizontal plane with a speed given by $v(x) = \alpha/x$, where x is its distance from the origin and α is a positive constant. Find the force $F(x)$ to which the particle is subject.

3. A gun is fired straight up. Assuming that the air drag on the bullet varies quadratically with speed, show that the speed varies with the height according to the equations

$$\begin{aligned} v^2 &= Ae^{-2kx} - \frac{g}{k} \qquad \text{(upward motion)} \\ v^2 &= \frac{g}{k} - Be^{2kx} \qquad \text{(downward motion)} \end{aligned}$$

   in which A and B are constants of integration, g is the acceleration of gravity, and $k = c_2/m$ where c_2 is the drag constant and m is the mass of the bullet. (*Note:* x is measured positive upward, and the gravitational force is assumed to be constant.)

4. The force acting on a particle of mass m is given by

$$F = kvx$$

   in which k is a positive constant. The particle passes through the origin with speed v_0 at time $t = 0$. Find x as a function of t.

5. Show by direct calculation that $\langle \sin^2(\omega t) \rangle = \frac{1}{2}$, where the time average is taken over any complete period, $t_1 \le t \le t_1 + 2\pi/\omega$.

   Show also that $\langle \sin(\omega t)\cos(\omega t) \rangle = 0$ when the average is over a complete period.

6. A piston executes a simple harmonic motion with an amplitude of 0.1 m. If it passes through the center of its motion with a speed of 0.5 m/s, what is the period of oscillation?

7. (a) A particle travels along a straight line with constant acceleration 1 ms^{-2} for 1 s, and then with acceleration of -1 ms^{-2} for 1 s. Assuming zero initial velocity, use `eXtrema` to plot $a(t)$, $v(t)$, and $x(t) - x(0)$. Three stacked plots aligned vertically would illustrate the relationship between the three graphs.

   Hint: Generate a time vector `t`, then use `eXtrema's`'s ability to include conditional operations in algebraic expressions to define `a` as `a=1.0*(t<1.0)+(-1.0)*(t>=1.0)`. This technique will come in handy later on in one of the lab experiments. Use numerical integration and differentiation (use the help facility to find out about the numerical functions `integral()` (and also `deriv()`) to get $v(t)$, and $x(t) - x(0)$. You may need to use non-default settings for `integral()`.

   (b) What is the maximum velocity reached during the motion, and when did the particle have its maximal velocity? What is the total distance travelled?

   (c) Separately, plot v as a function of x. What kind of a function is represented by this plot? Try to guess at the functional form, then confirm your guess algebraically. Overlay the analytical result as a curve of different line type and colour.

   (d) Modify your code to illustrate a similar relationship between $a(t)$, $v(t)$, and $x(t) - x(0)$ for another example of $a(t)$ of your own choosing.