

Assignment No. 2

Physics 2P20

Due October 3, 2024, 10:30

- (1) **1.** *EFTS*: Ex.5, the two-pulley system (K&K2 Pr. 2.8). Complete the solution started in class to show that

$$\ddot{x}_m = \frac{2M - m}{4M + m} g.$$

- (2) **2.** A particle of mass m moves along a frictionless, horizontal plane with a speed given by $v(x) = \alpha/x$, where x is its distance from the origin and α is a positive constant. Find the force $F(x)$ to which the particle is subject.
- (4) **3.** A gun is fired straight up. Assuming that the air drag on the bullet varies quadratically with speed, show that the speed varies with the height according to the equations

$$v^2 = Ae^{-2kx} - \frac{g}{k} \quad (\text{upward motion})$$
$$v^2 = \frac{g}{k} - Be^{2kx} \quad (\text{downward motion})$$

in which A and B are constants of integration, g is the acceleration of gravity, and $k = c_2/m$ where c_2 is the drag constant and m is the mass of the bullet. (*Note*: x is measured positive upward, and the gravitational force is assumed to be constant.)

- (2) **4.** The force acting on a particle of mass m is given by

$$F = kvx$$

in which k is a positive constant. The particle passes through the origin with speed v_0 at time $t = 0$. Find x as a function of t .

- (1) **5.** Show by direct calculation that $\langle \sin^2(\omega t) \rangle = \frac{1}{2}$, where the time average is taken over any complete period, $t_1 \leq t \leq t_1 + 2\pi/\omega$.

Show also that $\langle \sin(\omega t) \cos(\omega t) \rangle = 0$ when the average is over a complete period.

- (2) **6.** A piston executes a simple harmonic motion with an amplitude of 0.1 m. If it passes through the center of its motion with a speed of 0.5 m/s, what is the period of oscillation?

- (5) **7.** (a) A particle travels along a straight line with constant acceleration 1 ms^{-2} for 1 s, and then with acceleration of -1 ms^{-2} for 1 s. Assuming zero initial velocity, use **eXtrema** to plot $a(t)$, $v(t)$, and $x(t) - x(0)$. Three stacked plots aligned vertically would illustrate the relationship between the three graphs.

Hint: Generate a time vector \mathbf{t} , then use **eXtrema**'s ability to include conditional operations in algebraic expressions to define \mathbf{a} as $\mathbf{a} = 1.0 * (\mathbf{t} < 1.0) + (-1.0) * (\mathbf{t} \geq 1.0)$. This technique will come in handy later on in one of the lab experiments. Use numerical integration and differentiation (use the help facility to find out about the numerical functions `integral()` (and also `deriv()`) to get $v(t)$, and $x(t) - x(0)$. You may need to use non-default settings for `integral()`.

- (b) What is the maximum velocity reached during the motion, and when did the particle have its maximal velocity? What is the total distance travelled?
- (c) Separately, plot v as a function of x . What kind of a function is represented by this plot? Try to guess at the functional form, then confirm your guess algebraically. Overlay the analytical result as a curve of different line type and colour.
- (d) Modify your code to illustrate a similar relationship between $a(t)$, $v(t)$, and $x(t) - x(0)$ for another example of $a(t)$ of your own choosing.