

Experiment 10

Collisions and Conservation Laws

10.1 Elastic collisions

Conservation of momentum

Consider a collision of two objects with masses m_1 and m_2 , and with velocities $\vec{v}_1$ and $\vec{v}_2$ *before* the collision, and velocities $\vec{v}_1'$ and $\vec{v}_2'$ *after* the collision. To *predict* the final velocities, one would need to have a complete knowledge of the physical characteristics of the objects (size, shape, *etc.*) and of the geometry of the interaction. However, it is practical to measure all of the relevant quantities and then check to see whether the values are reasonable.

Conservation of momentum implies that

$$\vec{p}_1 + \vec{p}_2 = \vec{p}_1' + \vec{p}_2' . \quad (10.1)$$

Rearranging Equation 10.1 in terms of the changes in the momentum $\Delta\vec{p}_1 = m_1\Delta\vec{v}_1$ and $\Delta\vec{p}_2 = m_2\Delta\vec{v}_2$ of the two objects shows that the net change will be zero and that these vectors will be oriented anti-parallel to one another:

$$\vec{p}_1' - \vec{p}_1 + \vec{p}_2' - \vec{p}_2 = 0 \quad \rightarrow \quad \vec{p}_1' - \vec{p}_1 = -(\vec{p}_2' - \vec{p}_2) \quad \rightarrow \quad m_1\Delta\vec{v}_1 = -m_2\Delta\vec{v}_2 \quad (10.2)$$

Vector Equation 10.2 can be reduced to a scalar equation for the magnitudes of the vector changes:

$$m_1|\Delta\vec{v}_1| = m_2|\Delta\vec{v}_2| , \quad (10.3)$$

or, rearranging Equation 10.3 as a ratio of masses m_1 and m_2 :

$$\frac{m_1}{m_2} = \frac{|\Delta\vec{v}_2|}{|\Delta\vec{v}_1|} = \frac{|\vec{v}_2' - \vec{v}_2|}{|\vec{v}_1' - \vec{v}_1|} . \quad (10.4)$$

Thus, measuring the masses of the two pucks and the velocities of the pucks before and after the collision to see if Equation 10.4 is satisfied is an experimental test of the law of conservation of linear momentum.

Conservation of energy

It is also of interest to know whether kinetic energy $K = \frac{1}{2}mv^2$ is conserved during a collision. The “restitution factor” $\mathcal{R} = K'/K$ is defined as the ratio of the total kinetic energy K' after the collision to the total kinetic energy K before; for a perfectly *elastic* collision, $\mathcal{R} = 1$, and

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 . \quad (10.5)$$

In an *inelastic* collision some of the kinetic energy may be transformed into heat and sound energy during the collision, or there could be frictional forces acting on the masses during the interaction, and so $\mathcal{R} < 1$. In principle, in a *superelastic* collision, the total translational kinetic energy may even increase ($\mathcal{R} > 1$), for example if some *rotational* kinetic energy imparted onto the pucks before the collision transfers into translational kinetic energy, or if additional energy is released by the collision itself (*e.g.*, a collision of two spring-loaded mousetraps).

Note that $\mathcal{R}$ depends on the square of the velocity and hence will be very sensitive to variations in v ,

$$\mathcal{R} = \frac{K'}{K} = \frac{m_1 v_1'^2 + m_2 v_2'^2}{m_1 v_1^2 + m_2 v_2^2}. \quad (10.6)$$

10.2 Experimental procedure

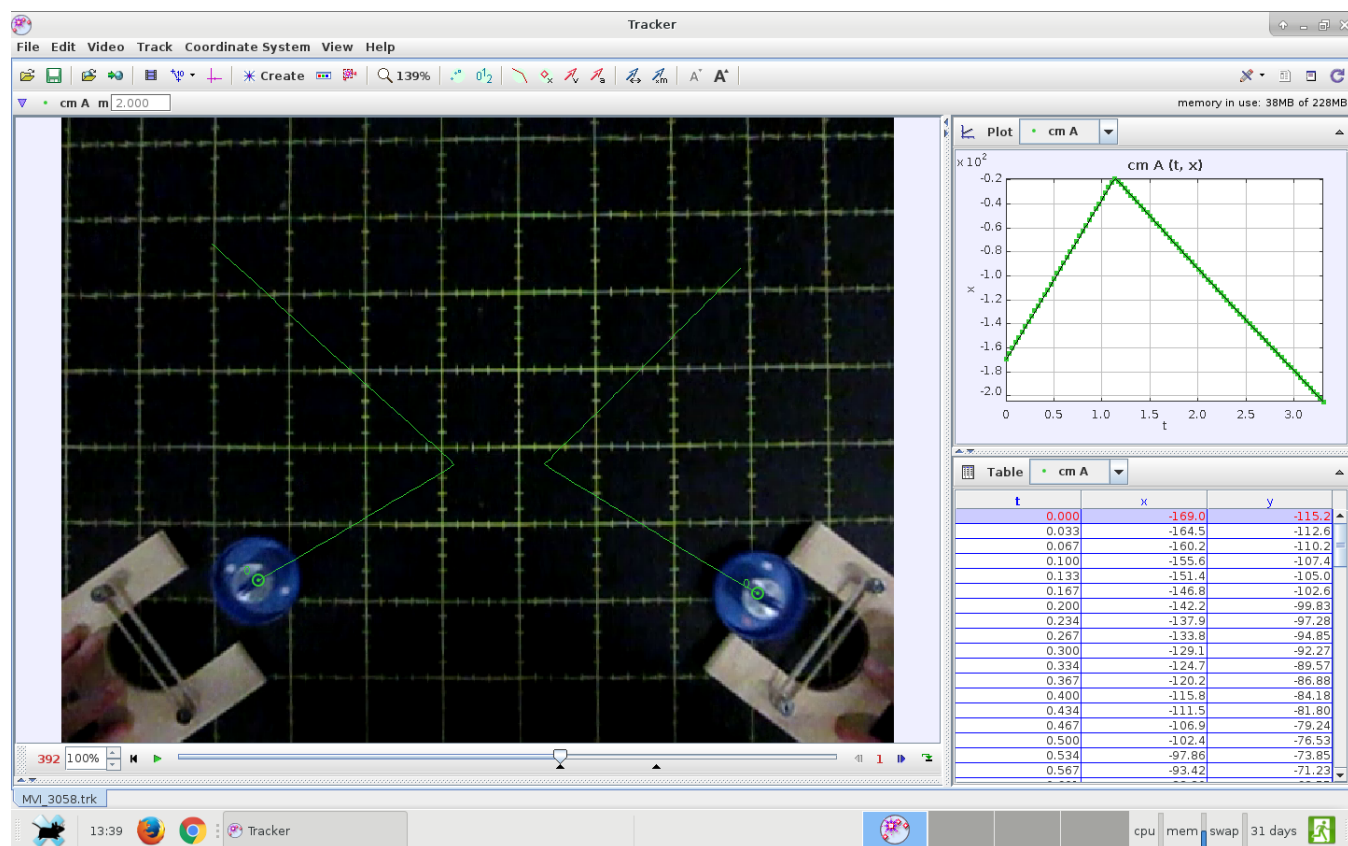


Figure 10.1: Trails left by moving pucks

The equipment consists of an air table that discharges streams of air from a series of closely spaced holes on its flat surface. Two plastic pucks float on this cushion of air with little friction. The pucks are launched and made to collide. An overhead camera records a movie of the collision. Software called **Tracker** is used to make a frame-by-frame analysis of the movie. For each frame, the software determines the position of the two pucks and records their coordinates in a table. These dots map the trajectory of the two pucks in time, as shown in Figure 10.1. From this data, vectors of the velocities before and after the collision are determined for each puck.

! To start, use a digital scale to record the masses of the two pucks.

Record a video of the collision

- ❗ In PhysTks, select **Hardware** → **USB camera**.
- ❗ Click **Start video stream**; a window should open to display the video from the camera situated above the air table. Verify that the table is framed evenly within the displayed picture. You can adjust the brightness of the image with the scrollbox above the video frame.
- ❗ Place the pucks on the air table and turn on the air until the pucks begin to float freely, then, if necessary, level the frame by adjusting the height of the two legs on the side until the pucks float in place.
- ❗ Place the launchers in adjacent corners of the frame. Load the pucks and press the trigger to release them toward the centre of the air table, where they will collide. Do several practice runs.

While *any* collision is theoretically valid, for ease of analysis your collision should be fairly symmetric as in Figure 10.1 and take about 2–3 seconds to complete. Small initial velocities will cause the pucks to slow down due to friction and yield unsatisfactory results.

- ❗ To record the collision, have one person press the **Capture video** button and another release the pucks. The recording ends automatically after about 5 seconds.
- ❗ Review your collision video by clicking on the **Review video** button. Repeat if necessary, making sure that the white dots on the pucks remain fairly sharp throughout, as these are the objects that the software will track.

Also, if the pucks are observed rotating during the collision, this may affect the results, and it is best to re-record another run.

Every student should record an individual video.

Analysis of the video with the Tracker software

The process is described in a slide show that can be found on the class website, <https://www.physics.brocku.ca/Courses/2P20/Tracker/>.

To start, you need to calibrate the video frame so that **Tracker** coordinate values correspond to the physical locations on the air table. This can be done using the 5-cm grid lines on the air table as follows:

- click **Track**, **New**, **Calibration Tools**, **Calibration Points**, then shift-click on intersecting grid lines about 15 cm (3 divisions) below and 15 cm to the left of the grid centre and enter -0.15 for the x and y coordinates of point 1;
- move 30 cm (6 divisions) up and 30 cm to the right of point 1, shift-click and enter 0.15 for the x and y coordinates of point 2.

The xy -axes should now be centered between these two reference points. You can review or change the coordinates of these points by clicking on them.

The next step is to trim the video clip to include only the portion of the movie that represents the collision you are going to analyse:

- below the progress scrollbar in the viewer window, adjust the left up arrow to advance the video, frame-by-frame, to the start of the collision sequence, when the pucks have left the launchers and both white dots are visible;
- adjust the right up arrow to set the end of the collision sequence, before the pucks hit the edge of the air table.

- Use the cue controls to review the movie and adjust the start and end frames, if necessary.

The actual centre of each puck is elevated from the air table surface, causing its position to be distorted in the video image as the puck moves around the field of view of the camera. To avoid this problem, the two white dots on the flat surface of the puck are used to define a virtual centre of mass for each puck that will be tracked accurately.

- Reset the movie to the start of the selection, then click **Track**, **New**, **Point Mass**. Note that **mass A** is added to the task bar above the video.
- Press **Ctrl-Shift**; a cross-hair appears. Move it over one white dot on Puck 1 and right-click. A square outline appears centered on the dot and the autotracker window opens. Visually check that the **Template** and **Match** graphics agree, then press **Search**. A frame-by-frame progression of the dot will be shown for the selected range to be analysed.
- Repeat the above steps for a **mass B** and the opposite dot on Puck 1.
- Define a centre of mass for Puck 1 by clicking **Track**, **New**, **Centre of Mass**. An entry window opens for **cm A**. Check **mass A** and **mass B** then **OK**. The midpoint of masses **A** and **B** on Puck 1 is now highlighted and tracked.
- Repeat these steps for Puck 2, adding **mass C,D** and their centre of mass **cm B**.

You can disable a trail by clicking on the label in the task bar. In the options by window that opens, uncheck the **visible** option.

Analysis of puck trajectories:

A few points around the time of collision may appear erratic. Make a note of the times of the beginning and end of the collision itself, as you will need to divide the data set for each puck into two clearly defined subsets, one for before and one for after the collision.

Tracker software does have options to display and/or save velocities of the two pucks. The **Table view** allows the display of a variety of parameters such as x, y, v_x, v_y, θ .

Use **File**, **Export**, then **Data file** to save trace data as a plain text file for each of **cm A** and **cm B**) and to continue your analysis in **eXtrema**. Use **eXtrema** to perform all the required calculations; this will prevent arithmetic errors.

- ❗ In **eXtrema**, the function **iclose(...)** will prove useful; since the x -component of each velocity in your collision of approximately equal masses changes sign, you could find the point in the data set where v_x is close to zero; since there are probably a few erratic points around the time of the collision, it's probably a good idea to skip a few points on either side. Here's a sample **eXtrema** syntax that could be used:

```
...
i0=iclose(v1x,0)
V1_before=sqrt(mean(v1x[1:i0-5])^2+mean(v1y[1:i0-5])^2)
V1_after=sqrt(mean(v1x[i0+5:#])^2+mean(v1y[i0+5:#])^2)
...
```

The end result of this first step of the analysis is to obtain and record the values listed in Table 10.1.

	Puck 1		Puck 2	
	value	$\pm$	value	$\pm$
mass, $m =$				
<i>before collision</i>				
$v_x =$				
$v_y =$				
$v = \sqrt{v_x^2 + v_y^2} =$				
$\theta = \arctan(v_y/v_x) =$				
<i>after collision</i>				
$v'_x =$				
$v'_y =$				
$v' = \sqrt{v'^2_x + v'^2_y} =$				
$\theta' = \arctan(v'_y/v'_x) =$				

Table 10.1: Puck masses and velocity components

- ❗ Use the coordinate/velocity data for the centre-of-mass of each puck to calculate the velocity change vectors

$$\Delta \vec{v}_1 = \vec{v}'_1 - \vec{v}_1 \quad \text{and} \quad \Delta \vec{v}_2 = \vec{v}'_2 - \vec{v}_2 .$$

It is probably the easiest to perform vector subtraction in Cartesian coordinates:

$$(\Delta \vec{v}_1)_x = v'_{1x} - v_{1x} \quad \text{and} \quad (\Delta \vec{v}_1)_y = v'_{1y} - v_{1y} , \quad (10.7)$$

and similarly for the second puck.

eXtrema syntax fragment could be something like this:

```
DeltaV1x=mean(v1x[i0+5:#])-mean(v1x[1:i0-5])
DeltaV1y=mean(v1y[i0+5:#])-mean(v1y[1:i0-5])
...
```

- ❗ According to Equation 10.2, the change in momentum of one puck is equal and opposite to that of the other puck. Compare the lengths and orientation (angle) of vectors $(\vec{v}'_2 - \vec{v}_2)$ and $(\vec{v}'_1 - \vec{v}_1)$.

Remember that the periodicity of $\tan()$ is π , not 2π , and this leads to some ambiguity in calculating its inverse using $\arctan()$. In **eXtrema**, the functions **atan()** and **atan2()** are slightly different, and one needs to be careful in interpreting the reported results, as additional considerations may be needed to determine the quadrant of the angle, assumed to be in radians, reported by the function you use. See the help menu. A sketch is always helpful. The equivalent functions **atand()** and **atan2d()** expect the angle to be in degrees.

- ❓ Is momentum conserved in the collision? Review the four sets of velocity data. Is there a systematic decrease of velocity in time, i.e. is `deriv(t,v)<0`?

- ❗ Use Equation 10.4 to calculate a value and error for the experimental mass ratio of the pucks, and compare it to the ratio m_1/m_2 obtained directly from the scale measurements of m_1 and m_2 . All calculations should be accompanied by proper error analysis; to refresh the error propagation rules, please review Section 1.4.
- ❓ Do the results agree within experimental error?
- ❗ Use Equation 10.6 and the directly measured m_1 and m_2 values to calculate $\mathcal{R}$ (and its error estimate).
- ❓ Can this collision be called elastic?

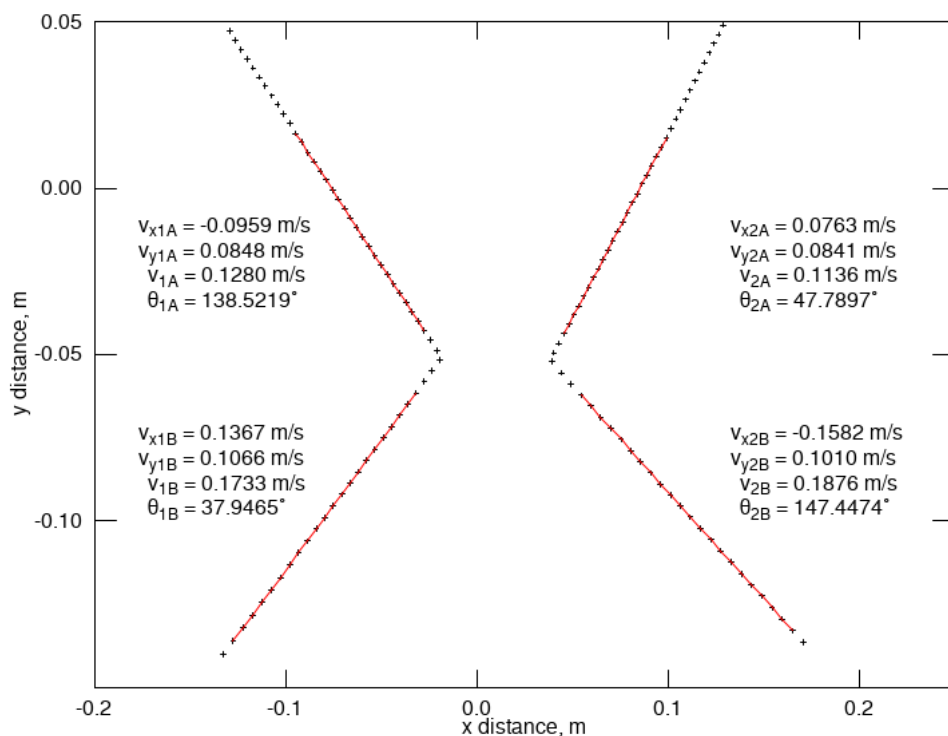


Figure 10.2: Representative Tracker collision data rendered and analysed in eXtrema