Experiment 8

Kater's Pendulum

Local value of g can be quite precisely measured using a pendulum of known length. In practice, however, it is not easy to get a very precise measurement of the location of the center of mass of an extended object. Several clever schemes exist to overcome that by adjusting the arrangement of multiple masses on a pendulum so that only the distance between two pivot points is needed, and the exact location of the masses is not necessary, as long as the two periods of oscillations about the two pivot points are identical. A commonly used design is Kater's pendulum.

8.1 Introduction

The period T of a physical pendulum is given by

$$T = 2\pi\sqrt{\frac{I}{mgd}}\,, \tag{8.1}$$

where I is the moment of inertia about to the axis of rotation (AR), or the suspension point of the pendulum; m is the total mass of the pendulum; g is the acceleration due to gravity; and d is the distance between the centre of mass (CM) of the pendulum and the AR. This classic result is derived under the so-called "amall-angle approximation", $\sin\theta \cong \theta$. When the amplitude of oscillations is such that this approximation ceases to be valid, then the expression for T requires modification:

$$T = 2\pi\sqrt{\frac{I}{mgd}}\left[1 + \frac{1}{2^2}\cdot\sin^2\left(\frac{\theta_m}{2}\right) + \frac{1}{2^2}\cdot\frac{3^2}{4^2}\cdot\sin^4\left(\frac{\theta_m}{2}\right) + \cdots\right]\,, \tag{8.2}$$

where θ_m is the maximum angular displacement. This improves the precision of the calculation, so that in principle, measuring T for a known m and d would yield an excellent way of obtaining the value of local g with high accuracy. The problem, however, lies with the "known" d, because for any realistic pendulum the position of the center of mass is not accurately known (the mass of the rod on which the pendulum bob sits cannot be neglected).

Kater's pendulum[1] to the rescue! This ingenious device (shown schematically in Fig. 8.1) consists of a bar with two fixed knife edges, AR_1 and AR_2; two different masses m_1 and m_2 can be clamped to the bar. The positions of m_1 and m_2 along the bar are adjusted until the period of oscillations T_1 around AR_1 is

[1] N. Feather, *An Introduction to the Physics of Mass, Length and Time*. The University Press; 1st edition, 1962. Pp. 187–194.

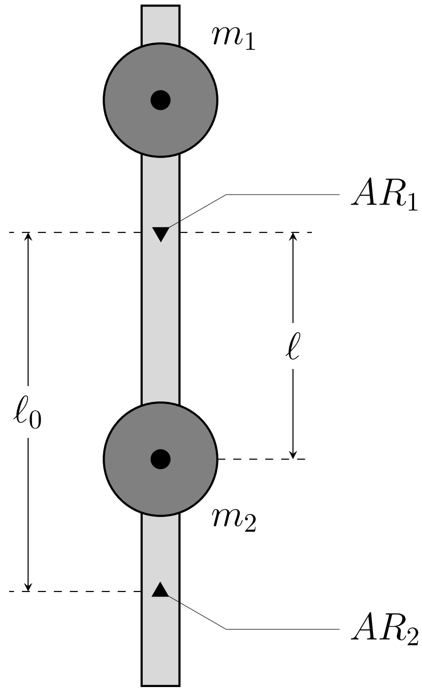

Figure 8.1: Dimensions of Kater's Pendulum

equal to the period of oscillation T_2 around AR_2. In this condition we then have

$$T_1 = T_2 = \sqrt{\frac{4\pi^2 \ell_0}{g}}\,, \tag{8.3}$$

where ℓ_0 is the distance between the knife edges. In other words, one need not know the precise position of the center of mass, but only the distance between the two points of suspension (the two knife edges), and a precise determination of g becomes possible with only one measurement of distance between two physically accessible points.

8.2 Experimental procedure

(!) In `PhysTks`, select Hardware, IOLab sensors and check `Magnetometer`. The 3-axis magnetometer detects variations in magnetic field resulting from the motion of a magnet attached to the ends of the pendulum. The sensor is located under the **M** symbol on a corner the `iOlab`.

Likely, only one axis will produce a decent waveform; adjust the position of the `iOLab` remote to tweak the shape so it resembles a sine wave.

Hint: be sure to `select y#` to match the axis that you intend to fit to.

(!) Using the calipers and given that the distance between the pendulum holes is 10.0 mm and the hole diameters are 5.0 mm, determine the distance ℓ_0 between the knife edges AR_1 and AR_2.

(!) With the calipers, carefully measure the distance from the *centre of the fixed mass* m_1 to the *knife edge* AR_1. Do not adjust the m_1 position.

(!) Setup mass m_2 and record the distance ℓ from it's *centre of mass* to the *knife edge* AR_1.

- (!) Set up Kater's pendulum, making sure it swings freely. The pendulum should be able to swing for several minutes without any appreciable decay. Check also that the pendulum swings in a plane.

- (!) With small θ_m and AR_1 as the pivot point, measure T_1 a few times by fitting to the resulting sine wave.

  Hint: organize yourself to prevent duplication of effort and the introduction of errors. Don't waste time writing data on paper; you will then have to manually copy this content to a file. Record your data directly to a file that can be read and analysed with `eXtrema`.

- (!) Now reverse the pendulum, so that it pivots on knife edge AR_2 and measure T_2. Unless you are very lucky, T_1 and T_2 will be appreciably different.

- (!) Move m_2 to another hole without altering the arrangement of the washers, then record this new ℓ and measure again the corresponding T_1 and T_2.

- (!) Repeat this procedure for at least a 20-25 points to obtain a series of values for ℓ that span the distance between AR_1 and AR_2, as shown in Fig. 8.2.

- (!) Use `eXtrema` to plot $T_1(\ell)$ and $T_2(\ell)$ on the same graph. The intersection of these two curves indicates the values of ℓ for which T_1 is approximately equal to T_2.

- (!) Fit the two curves and find the points of intercept for T_1 and T_2. Use a polynomial that best agrees with the trend of your data.

- (!) For the two points of intercept, use their T and ℓ values to calculate g from Equation 8.3. Compare the two values of g; do they agree within their experimental errors?

  The inherent accuracy of this experiment is quite high. Give a complete error analysis, including a discussion of whether your measured T values should be corrected for large oscillation amplitudes θ_m. Use Equation 8.2 to prove this.

As always, express your results properly rounded according to their associated errors and remember to include their respective physical units, if any.

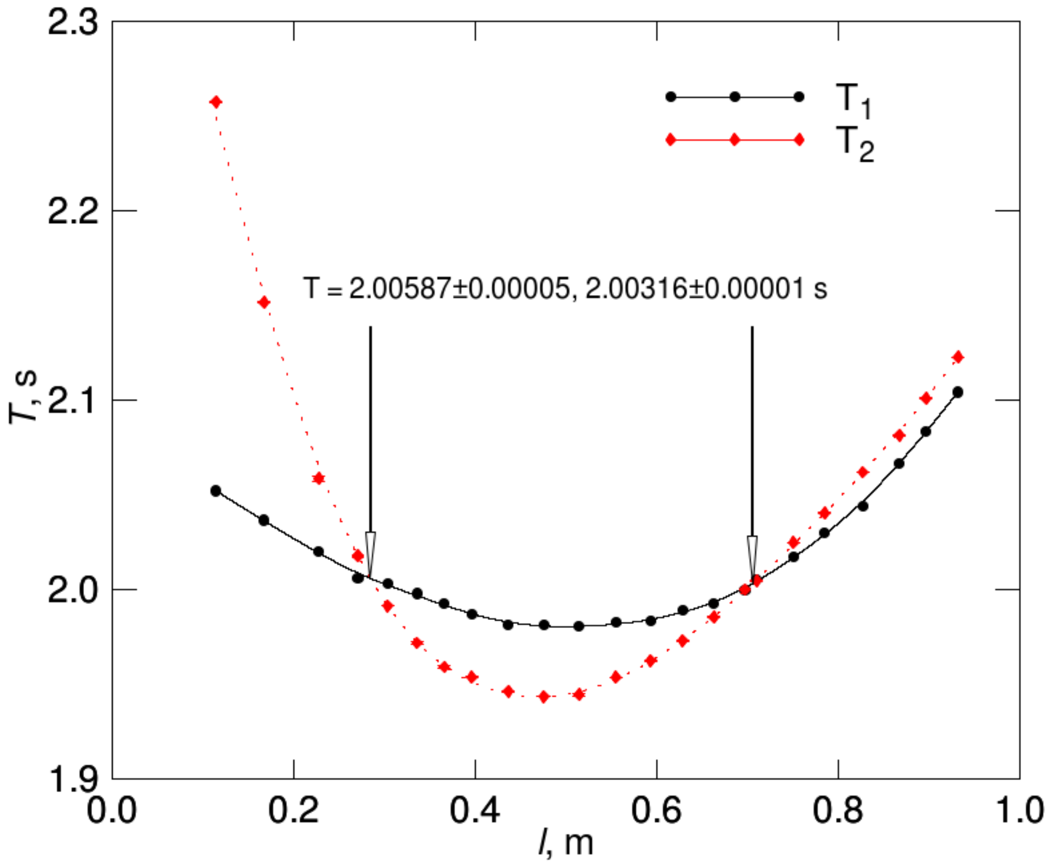

Figure 8.2: An example of determination of the acceleration due to gravity from a Kater's pendulum experiment. The interpolation method was used to find the two intercept points where $T_1 = T_2$. For $\ell_0 = 0.9975$ m, the result was found to be $g = 9.801 \pm 0.013$ m/s^2, at the location where the known value was $g = 9.8038$ m/s^2.