Experiment 3

Damping and Anharmonicity

This is a continuation of the previous experiment, so the setup is very similar. This experiment is dedicated to exploring the imperfections of the SHO, through measuring the effects of damping and of deviations from harmonicity.

3.1 Damped harmonic oscillations

You may have already noticed that there was a slight loss of energy in the motion of a glider on the air track. It is not a perfectly friction-free apparatus, since a weak viscous damping is already provided by the air resistance to the moving glider. It can be increased in a controlled manner by attaching strong magnets to the sides of the glider: since the the track is non-magnetic there is no direct interaction with it, but the eddy currents induced in the (non-magnetic, but conducting) metal sides of the air track by the *moving* magnets provide the interaction that dissipates the glider energy and slows it down. This retarding air-resistance-like force is proportional to the velocity v of the glider. The differential equation of motion is thereby modified:

$$m\frac{d^2x}{dt^2} = -kx - bv, \tag{3.1}$$

or

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = 0, \tag{3.2}$$

with $\gamma \equiv b/(2m)$ and $\omega_0^2 \equiv k/m$.

For a reasonably weak damping ($\gamma < \omega_0$) the motion of the glider is still an oscillation, but with a frequency $\omega_d (\neq \omega_0)$ (subscript d stands for "damped") and a steadily decreasing amplitude:

$$x(t) = Ae^{-\gamma t}\cos(\omega_d t + \varphi_0)\,, \tag{3.3}$$

with the amplitude A and the initial phase φ_0 determined by the initial conditions and the frequency of damped oscillations given by

$$\omega_d = \sqrt{\omega_0^2 - \gamma^2}. \tag{3.4}$$

Experimental procedure

- (!) Review the procedure from Experiment 2 to ensure the proper operation of the air track.
- (!) Setup the air track with the springs connected to the glider as in Figure 3.1.

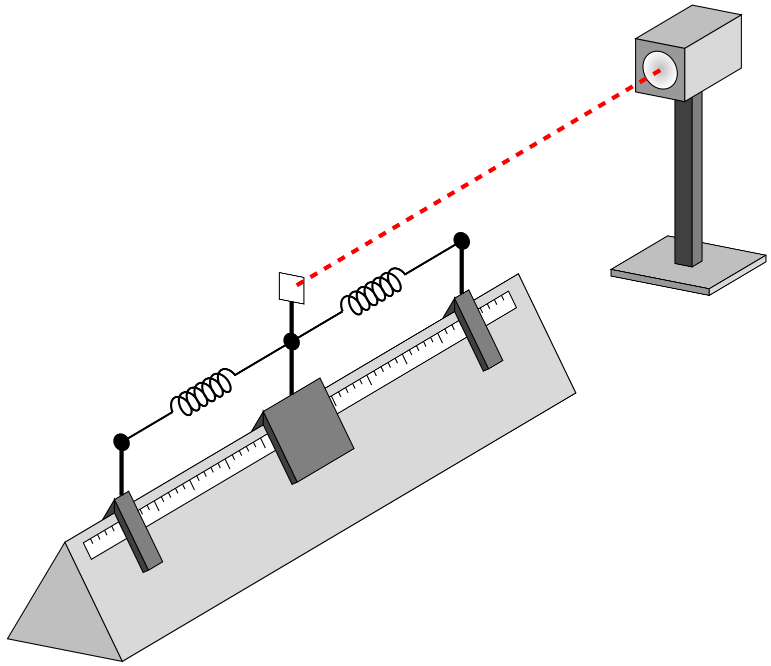

Figure 3.1: Overall arrangement for the damped harmonic oscillator

(!) With the magnet(s) attached to the top of the glider, determine the total mass of the system (glider+magnets) using a digital scale. Place the glider on the air track and determine ω_0. Take this measurement three or more times and calculate the standard deviation to determine the error $\sigma(\omega_0)$.

(!) Transfer the magnet(s) to the side(s) of the glider, in close proximity to the metal of the track, set the glider in a damped oscillatory motion, and measure $\omega_d \pm \sigma(\omega_d)$ over a few oscillations, taking damping into account during the fitting process. Then calculate γ from Equations 3.3–3.4.

3.2 Undamped anharmonic oscillations

When the restoring force on the glider is not exactly proportional to x, the glider still oscillates around $x = 0$, but the oscillations are not strictly harmonic, and amplitude and frequency are not independent. An example of an anharmonic oscillator is shown in Figure 3.2.

Experimental procedure

(!) Show that for the system in Figure 3.2 the restoring force (in the x-direction) on the glider is

$$F = -2kx\left(1 - \frac{\ell_0}{d\sqrt{1 + x^2/d^2}}\right) , \tag{3.5}$$

where ℓ_0 is the length of the relaxed spring, and $d > \ell_0$. Show that for small values of x/d this is approximately equal to:

$$F = -kx\left(2 - \frac{2\ell_0}{d}\right) - kx^3\left(\frac{\ell_0}{d^3}\right) . \tag{3.6}$$

You may want to consult the class notes; a similar derivation was done there.[1]

[1] A graduate-level course in classical mechanics may be required to fully analyse the anharmonicity, see for example `http://galileoandeinstein.physics.virginia.edu/7010/CM_22_Resonant_Nonlinear_Oscillations.html`

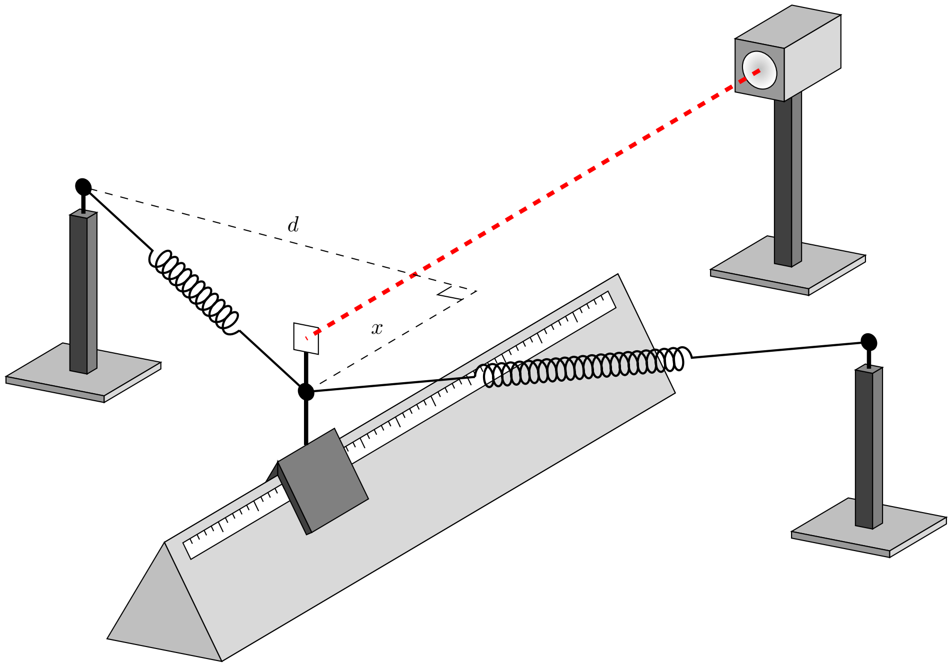

Figure 3.2: An arrangement of two springs that ensures anharmonicity of the oscillations.

(!) The resulting prediction of how oscillation frequency depends on the amplitude of the motion is

$$\omega = \omega_0 + \frac{3\beta}{8\omega_0} A^2 \,, \tag{3.7}$$

where ω_0 at low amplitudes has a characteristic increase with amplitude A.

Set up the oscillator as in Figure 3.2; make d long enough that the springs are only slightly stretched, to avoid sagging at the midpoint of travel of the runner. Carefully measure ω as a function of A by changing the initial displacement along x direction, or simply acquire a long data set and allow friction and air resistance to slowly reduce the amplitude of oscillations, and plot the results.

- Examine short segments for which the amplitude of oscillations can be considered approximately constant and determine the oscillation frequencies for each. Plot frequency as a function of amplitude, and determine β.

The following macro scans the data set of a well-behaved decaying oscillation and determines a series of (amplitude,omega) values by fitting sequential blocks of data of width di, always starting at a sine wave maximun to set the initial phase angle to approximately 1.5 radians. Currently, the block size and the initial guess for B must be manually adjusted. Try to add some code that will automatically determine reasonable estimates for these two parameters.

A useful function is imax(x) that returns the index where x is a maximum. Then t(imax(x)) returns the corresponding time. Similar index functions such as imin are available. Consult the help menus. Recall also that # refers to the last element of a vector, i.e. x[n:#].

```
read your_decaying_oscillation_data t x

scalar\vary A B C D
A=(max(x)-min(x))/2
B=5.5                      ! consider how to automate omega
C=1.5                      ! start of fit is always at max=Pi/2
D=(max(x)+min(x))/2

i=1
di=100                     ! consider how to automate block size to include 4-5 cycles
block=1

begin:                     ! labels, ending with : define branch target of goto

  findmin:
    if (x(i+1)<x(i)) then                 ! if true,execute to endif
      i=i+1                               ! if false, skip past endif
      if (i+di>=len(x)) then goto done    ! done if out of data
      goto findmin                        ! branch to findmin label
    endif

  findmax:
    if (x(i+1)>x(i)) then
      i=i+1
      if (i+di>=len(x)) then goto done    ! done if out of data
      goto findmax
    endif

  xx=x[i:i+di-1]                          ! create vectors of data block to fit
  tt=t[i:i+di-1]
  fit xx=A*sin(B*tt+C)+D
  Amp(block)=A                            ! store current fit results, will be
  Omega(block)=B                          ! used as initial guesses in next fit

  block=block+1
  i=i+di
  goto begin

done:

list Amp Omega
```

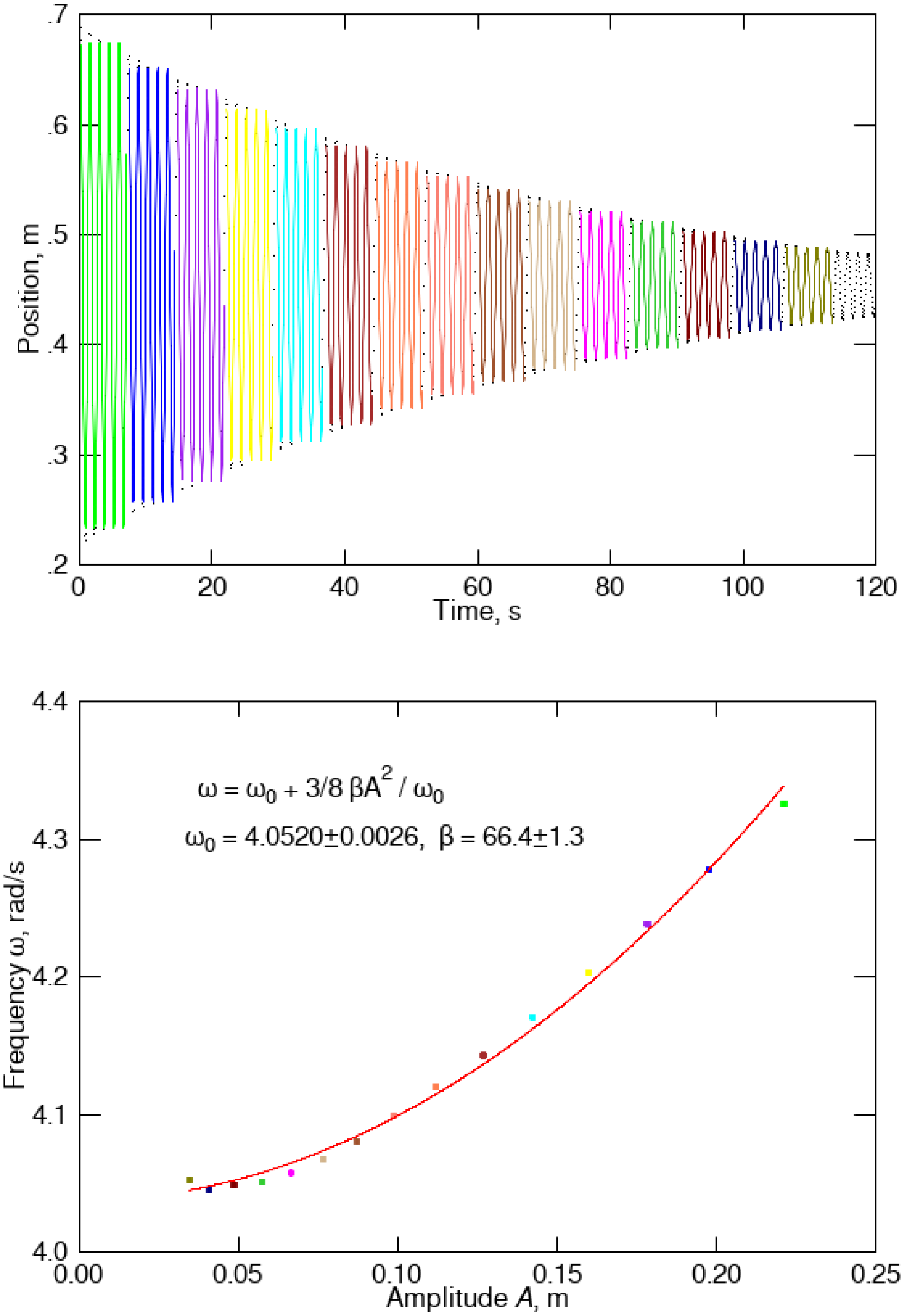

Figure 3.3: Representative plot of some anharmonic decay data, split into sections of approximately constant amplitude, each fitted to yield a series of amplitude/frequency pairs. These are then plotted and fitted to Equation 3.7.