Experiment 4

Resonance: Forced, Damped, Harmonic Oscillations

In this experiment, a driving periodic force keeps adding energy to the system, while damping keeps taking the energy out of the system. After a while, initially irregular motion settles into a steady-state, and it is the properties of this steady-state that are of interest here. Something very interesting happens when the frequency of the driving force comes close to the natural frequency of the oscillations of the system.

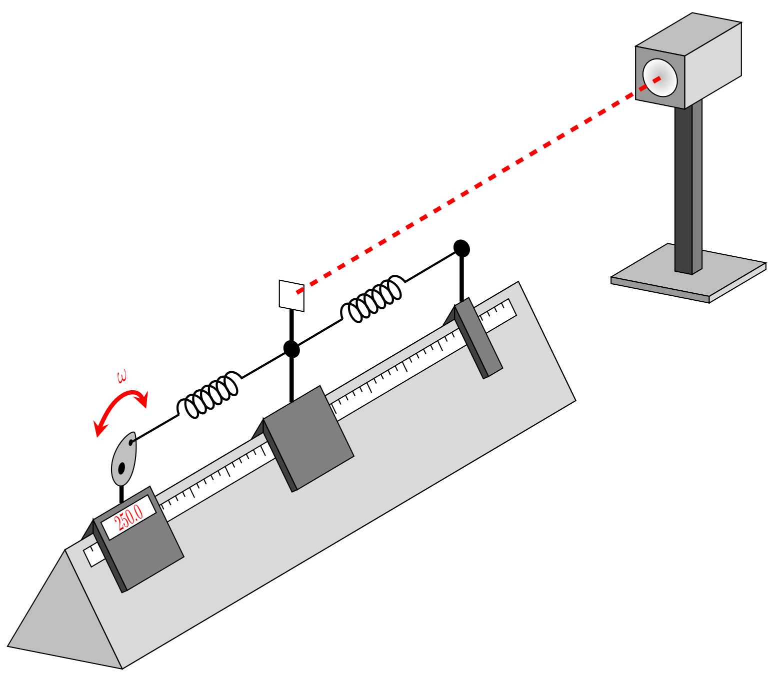

Figure 4.1: Overall arrangement for the resonance experiment on an air track

Experimental procedure

This is a continuation of the previous two experiments, so the setup is again very similar except a digitally controlled actuator is replacing a fixed point of attachment for one of the springs (see Fig. 4.1). The actuator arm at one end provides a continuous back-and-forth movement of the end of the spring, with about 1 cm amplitude at variable frequency.

(!) Review the procedure from Experiment 2 to ensure safe operation of the air track.

(!) Weight the glider with the digital scale.

(!) With the actuator turned off, start the glider oscillating to get an approximate value for the natural frequency w_0 of the system; use it to determine the range of frequencies that you will explore.

(!) Set the actuator in motion. The record of $x(t)$ will initially be an irregular motion (the transient), settling into a steady-state motion of the form $x = A\cos(\omega t + \varphi)$. As before, determine the frequency ω for this steady-state motion; amplitude and phase angle are given by

$$A = \frac{F_0}{m}\frac{1}{\sqrt{(\omega_0^2 - \omega^2)^2 + (2\gamma\omega)^2}} \tag{4.1}$$

$$\tan\varphi = -\frac{2\gamma\omega}{\omega_0^2 - \omega^2}. \tag{4.2}$$

(!) Repeat as necessary to measure A for a number of ω values, enough to generate a well-resolved graph of $A(\omega)$, as shown in Figure 6.2. Fit to Equation 4.1 to determine F_0, ω_0/m, and γ.

Hint: adjusting the driving frequency by a small amount will cause the resulting transient to settle more quickly to a steady-state amplitude.

(!) Show theoretically that:

1. $A_0 = A(\omega \to 0) = F_0/k_{\text{eff}}$.
2. A is a maximum for $\omega = \omega_r$, with

$$\omega_r^2 = \omega_0^2 - 2\gamma^2, \tag{4.3}$$

and

$$A_r = A(\omega = \omega_r) \cong \frac{F_0/m}{2\gamma\omega_0} = A_0\,\frac{\omega_0}{2\gamma}. \tag{4.4}$$

Determine A_0, A_r and ω_r from the resonance curve and determine F_0 and γ from them.

As you vary the actuator frequency ω, note qualitatively the relative phase $\Delta\varphi$ between the motion of the actuator arm and the air car. There is a significant change when ω increases from $\omega \ll \omega_r$ to $\omega \gg \omega_r$. Comment on that change in your lab report.

(!) The behaviour of a damped oscillator is determined by ω_0 and γ, which are often combined in a so-called "quality factor" $Q = \frac{\omega_0}{2\gamma}$.

Calculate Q for this oscillator. Q determines the "shape" of the resonance curve. If the two frequencies for which the amplitude of the response goes down to $1/\sqrt{2}$ of its maximum value (the "half-power point"),

$$A = \frac{1}{\sqrt{2}}A_r$$

are ω_+ and ω_-, then the half-width of the resonance curve, defined as $\Delta\omega = \omega_+ - \omega_-$, is approximately given by $\Delta\omega \cong 2\gamma$. Compare the predicted half-width of your resonance curve with the experimental value.

The name "quality factor" derives from the fact that the value Q roughly corresponds to the number of full oscillations that it takes for the damped (not driven) harmonic oscillator to stop oscillating.

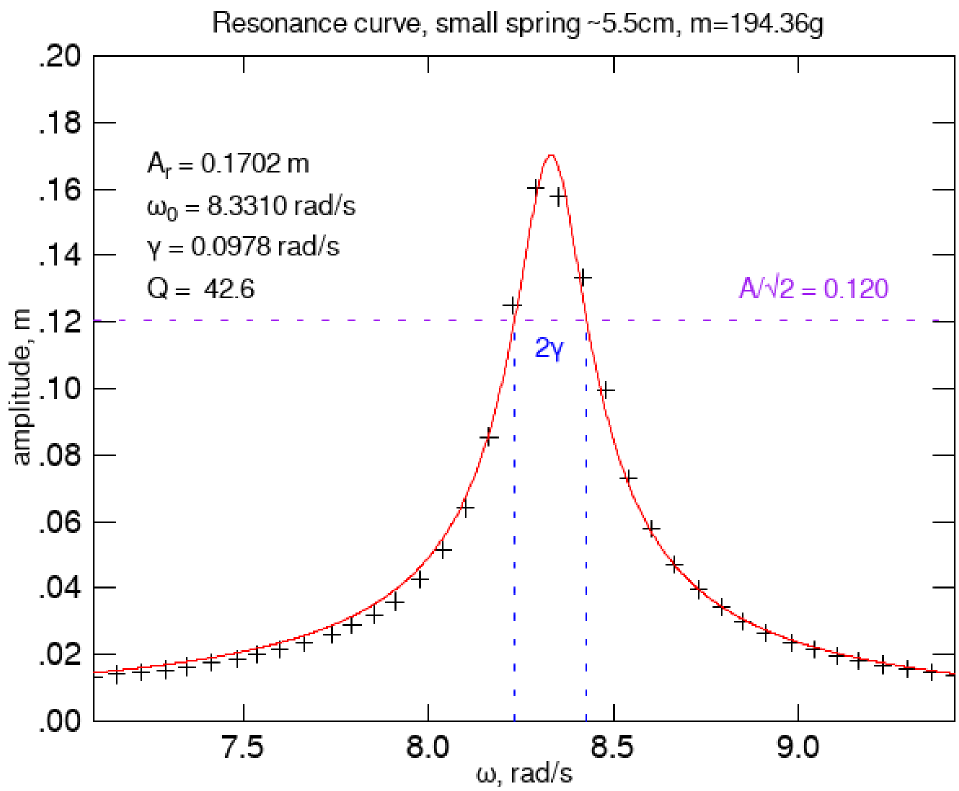

Figure 4.2: A typical resonance curve with the driving frequency resolved to 0.01 Hz