

Experiment 2

Simple Harmonic Oscillations

A linear air track is an excellent approximation to an almost friction-free environment, and it provides an opportunity to explore Newtonian motion in conditions that allow for highly simplified analysis of the data. A simple harmonic oscillator (SHO) motion will be explored.

2.1 Introduction

In this experiment, we will set up an oscillating system consisting of a glider (mass m) on the linear air track, connected to two stretched springs (spring constants k_1 and k_2). The arrangement is shown schematically in Figure 2.1. The position $x(t)$ of m will be recorded with an ultrasonic range finder controlled by **PhysTks**

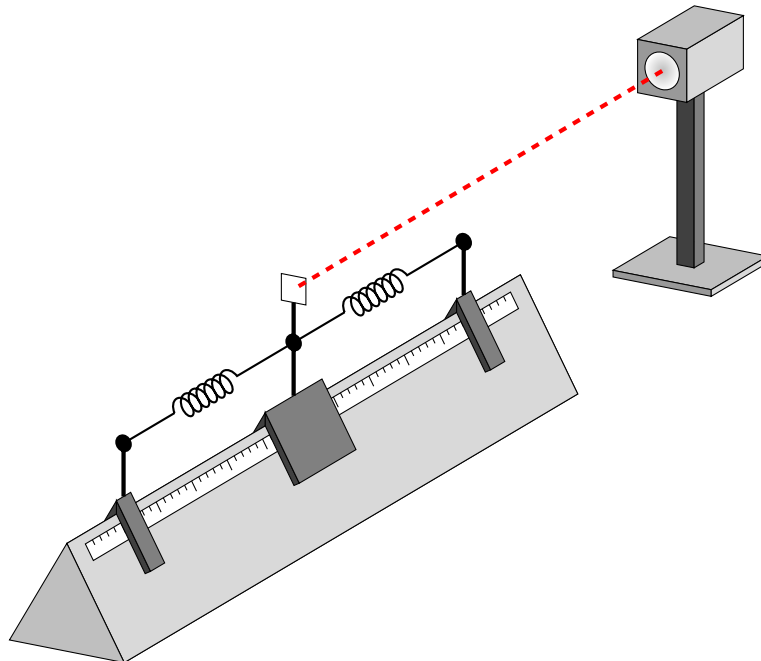


Figure 2.1: Overall arrangement for the SHO experiment on an air track

software, which will also be capable of performing a preliminary data analysis directly during the experiment. Full analysis can be performed during the lab report preparation, using **eXtrema**.

Experimental procedure

In preparing for the experiment, it might be useful to create a skeleton `.pcm` file for the analysis of the air track data; `PhysTks` saves $x(t)$ traces as two-column plain-text data files (the columns are t and x), so this is what your script should anticipate reading in.

- **Make sure to clean and level the air track prior to beginning the experiment for this and every subsequent lab that requires the use of the linear air track.**
- **Never drag gliders on the air track, this will damage both.**
- **Note that the minimum distance that the pinger can measure is 15 cm.**

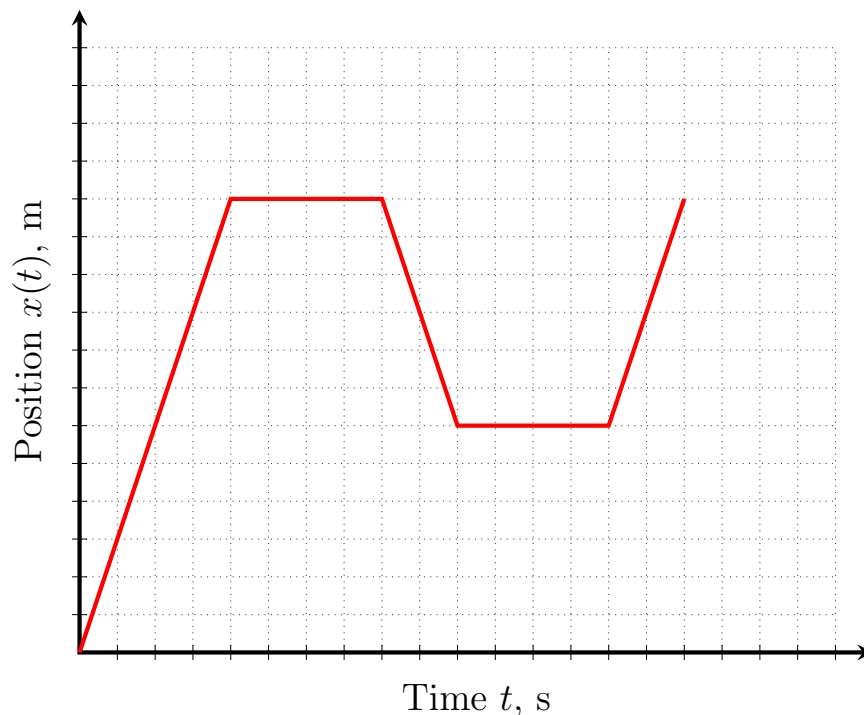


Figure 2.2: Using a hand to try to match this target $x(t)$ trace should give you a sense of what various kinds of motion look like. The rangefinder, a.k.a. pinger, measures the distance to your hand.

- ⓘ Using `PhysTks` and your hand generate an $x(t)$ trace similar to the one given in Figure 2.2. Feel free to repeat as many times as necessary, trying some simple motions first, then combine various sections of the motion into a single trace that looks close to Figure 2.2.
- ⓘ Set up a (clean) air glider on a (clean) air track, turn the air supply on, and level the air track until the floating air glider remains motionless. Verify at several positions along the track.
- ⓘ By applying a slight push to the glider, generate a trace of motions with a constant velocity. Using `eXtrema` determine the actual velocity (the slope of x vs. t) from the $x(t)$ trace.
- ⓘ Using a few sheets of folded paper on the track as a brake, generate a trace of a decelerating glider. Plot and analyze to determine the average acceleration a . Determine the time limits of the decelerated motion: when does braking first occur, when is the glider stationary.

The initial velocity is determined from the linear portion of the $x(t)$ trace, while the deceleration time is measured from the first signs of slowing until the glider is motionless. Determine those times approximately from the graph of the trace, and then try to refine them using `eXtrema`'s `fit` command. Note that it is possible to create a piece-wise defined function by using conditionals in the function definition by typing, for example `x=x0+v0*t+0.5*a*(t-t0)^2*(t>=t0)`, which corresponds to

$$x(t) = \begin{cases} x_0 + v_0 t, & t < t_0 \\ x_0 + v_0 t + \frac{1}{2} a (t - t_0)^2, & t \geq t_0 \end{cases}$$

and that the location of the switchover from one function to the other (t_0 above) can itself be a parameter of the fit in `eXtrema`, not just the coefficients x_0 , v_0 , and a .

A useful `eXtrema` trick is to determine the section of your data where the glider has stopped automatically as the area where the velocity has dropped below a certain threshold value. You may use this macro fragment, but figure out what these commands do (use Help) and explain it in the lab report:

```
points=13                ! data points to smooth over, try different values
v=deriv(t,savgol(2,points,x)) ! filter out noise, then calculate dx/dt
w=abs(v)>0.015           ! "moving" means |v|>1.5cm/s
xb=mean(1-w,x)          ! the baseline is where the glider is stopped
fit\weight w x-xb=x0+v0*(t-t0)+0.5*a*(t-t0)^2*(t>=t0)
```

2.2 Undamped harmonic oscillator

When the force F on the glider is proportional to the displacement x from its equilibrium position, then the glider will perform harmonic oscillations around $x = 0$. We then have for the force F ,

$$F = -kx,$$

for the potential energy U ,

$$U = \frac{1}{2} kx^2,$$

and for the differential equation of motion,

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0, \quad (2.1)$$

with $\omega_0^2 = k/m$. The motion is described by the solution to Equation 2.1,

$$x = A \cos(\omega_0 t + \varphi). \quad (2.2)$$

Amplitude A and phase angle φ depend only on initial conditions, and are independent of ω_0 .

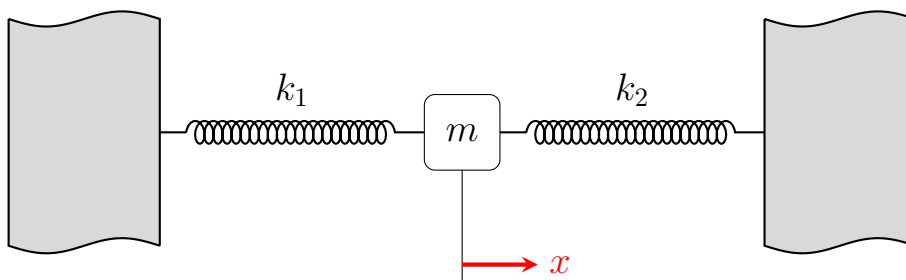
Experimental procedure

ⓘ Turn on the air supply, then setup the linear air track as shown in Figure 2.1.

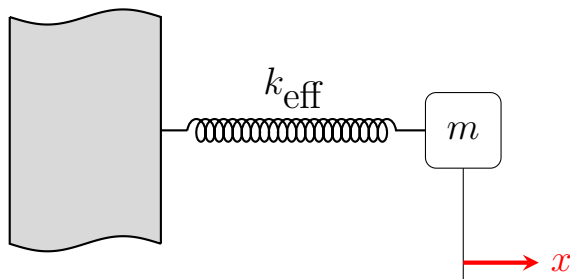
Separate the spring mounts so that the glider can move some 30-40 cm without a spring sagging. Do not over-stretch a spring past the elastic limit; the spring constant will change.

Check that there are no damping magnets on the glider and that the pinger target flag is perpendicular to the air track. The glider should be floating motionless, at equilibrium, on the air track.

- ⓘ Open **PhysTks** and select **LabPro** from the **Hardware** menu. Check that **Dig1** is selected to collect rangefinder data. Apply a gentle push to the the glider and monitor the resulting oscillatory motion over at least 4-5 complete oscillations. Adjust the rangefinder, if necessary, until you obtain a smooth sine wave.
 - ⓘ Fit the trace to a sine/cosine function to determine the frequency of the oscillations. Good initial guesses for the fit parameters are needed for the fits to converge; determine those through a careful inspection of the $x(t)$ plot.
 - ⓘ Repeat, using a different amplitude of the motion, or simply wait for the amplitude to decrease due to the air resistance and then repeat the measurement. Record and plot $\omega_0 \pm \sigma(\omega_0)$ of the oscillator as a function of amplitude; the high-amplitude limit is determined by the condition that neither of the two strings is sagging or touching the air track.
- At really high amplitudes and, therefore, high speeds of the glider, the air resistance may become high enough to affect the oscillation frequency; we will explore this phenomenon in Experiment 3.
- ⓘ Show theoretically that the system



is equivalent to the system:



and derive an expression for k_{eff} in terms of k_1, k_2 and any other experimentally relevant parameters. Use the digital scale to determine m . From ω_0 and m , calculate k_{eff} .

Rangefinder calibration

The pinger measures the time it takes for an emitted high-frequency sound pulse (ping) to reach the object, bounce off and return to the pinger. To convert this time-of-flight to a distance, the pinger uses a stored value for the speed of sound. Since the speed of sound in the lab may be slightly different due to environmental factors such as temperature, water vapour pressure, *etc.*, you can minimize any systematic error in your distance measurements by calibrating these to a known reference.

- ⓘ To do this, place a glider carefully on the track. Record the reading from the scale on the side of the track and the corresponding pinger average distance given in **PhysTks** . Repeat for other positions using the same index reference on the glider. Plot and fit these data pairs to an appropriate expression that you can use in **eXtrema** to improve your results for position, velocity and acceleration.