Experiment 5

Probing an Interaction Potential

Any interaction between objects can be described in terms of its so-called interaction potential, the energy required to bring two interacting objects together as a function of the distance between them. A famous example is the Lennard-Jones potential curve that models atomic interactions. A common way of probing repulsive interaction potentials is to send a probe particle with a certain initial kinetic energy into a target, and measure the distance of nearest approach that can be achieved, until a repulsive force turns the probe particle around and repels it. In this experiment, the potential of two interacting magnetic dipoles is explored on a linear air track in an almost friction-free environment.

5.1 Introduction

A glider of mass m with a strong magnet mounted on it is moving with a velocity $\vec{v}_0$ toward an oppositely-oriented stationary magnet. The force between the two magnets is repulsive and increases the closer the two magnets get, and at some point the glider will reverse its direction of motion. This point of nearest approach depends on m, v_0 and the strength of the magnetic interaction, as illustrated in Fig 5.1. This

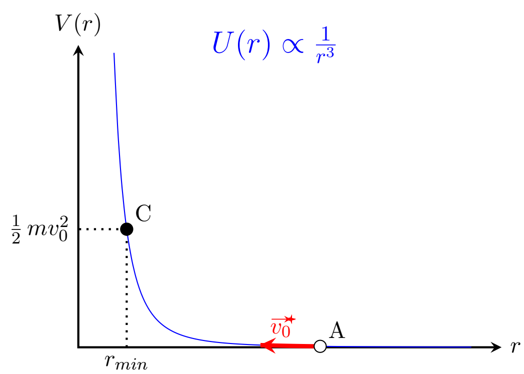

Figure 5.1: Probing the potential energy curve by sending energetic probe particles and measuring the distance of nearest approach.

strength is given by a potential function $U(r)$, with r as the separation of the two magnets. If the track is truly free of friction, then $U(r)$ is easy to determine: measure v_0 at a position, say at A, where $U(r)$ is negligibly small. If the turning point C is at position x_C, then conservation of mechanical energy gives

$$\text{total energy at A} = E(x = x_A) = E(x = x_C) = \text{total energy at C}\,.$$

This yields

$$\frac{1}{2}mv_0^2 + 0 = 0 + U(r) \tag{5.1}$$

where point A is assumed to be so far away that the interaction between the two magnets is assumed to be negligible, *i.e.* $U(\infty) = 0$.

The track is not completely friction-free, and the frictional loss in kinetic energy while the glider travels from A to C needs to be taken into account. Using the work-energy theorem,

$$W_m + W_f = \frac{1}{2}mv^2\Big|_{\text{at C}} - \frac{1}{2}mv^2\Big|_{\text{at A}} = -\frac{1}{2}mv_0^2\,, \tag{5.2}$$

where v_0 is the speed at A since the speed at C is zero. W_f is the work done by the friction forces, and W_m is the work done by the magnetic forces. Since the magnetic force is negligibly small at A (the two magnets are far apart),

$$W_m = -U(r)\,. \tag{5.3}$$

Calculation of W_f

The origin of the frictional force F_f is the friction in the air layer between the glider and the track. For small glider velocities v this force is proportional to v, producing an acceleration $a = -kv$, where k is a constant. For this type of acceleration, we know that velocity v and position x depend on time t as follows:

$$a = -kv \tag{5.4}$$

$$v = v_0 e^{-kt} \tag{5.5}$$

$$x = \frac{v_0}{k}\left(1 - e^{-kt}\right) \tag{5.6}$$

from which it immediately follows that

$$a = k^2 x - kv_0 \tag{5.7}$$

$$v = v_0 - kx. \tag{5.8}$$

Therefore, the work done by the frictional force while the glider moves from A to C is approximately[1] given by

$$W_f = \int_0^r \vec{F}_f \cdot d\vec{x} = m\int_0^r (k^2 x - kv_0)\,dx = m\left(\frac{1}{2}k^2 r^2 - kv_0 r\right). \tag{5.9}$$

Combining Equations 5.2, 5.3, and 5.9 gives $U(r)$:

$$U(r) = \frac{1}{2}mv_0^2 + \frac{1}{2}mk^2 r^2 - mkv_0 r. \tag{5.10}$$

We can determine k by measuring the inital velocity v_0 and the final velocity v_1 of the glider near point A where the magnetic force is still negligible; v_0 as it approaches C and v_1 after it has changed direction at C. The velocity thus decreases by $v_0 - v_1$ over a path length of $2(x_A - x_C)$. Then, from Equation 5.5 we have

$$k = \frac{v_0 - v_1}{2(x_A - x_C)}\,. \tag{5.11}$$

Therefore, all quantities in Equation 5.10 can be measured, and $U(r)$ can be determined.

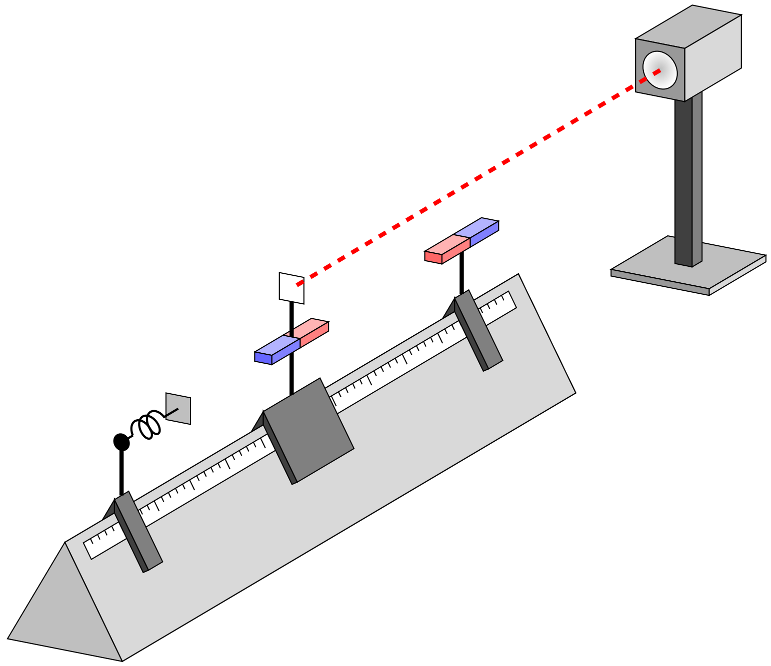

Figure 5.2: Experimental setup. The magnet on a glider probes the field of the magnet fixed at one end of the linear air track. The distance of closest approach is where the glider reverses direction, $v = 0$. The spring bumper at the far end reverses the direction of the glider to allow multiple "bounces".

5.2 Experimental procedure

The experimental setup is illustrated in Fig. 5.2. As usual, an ultrasonic rangefinder is used to monitor the velocity of the glider, from its initial value v_0 far away from the region of interaction, to the point of turnaround where $v = 0$. Below, the references to an "indexing block" are to an additional reflecting surface that you can place on the air track to measure the x-coordinates of some specific locations along the track.

(!) Weigh the glider to obtain m.

(!) Review the procedure from Experiment 2 to ensure safe operation of the air track, then carefully level the air track. Verify that the air is turned off.

(!) Place the indexing block against the fixed magnet then acquire a few data points to determine the offset d_1 between the rangefinder sensor and the face of the fixed magnet.

(!) Carefully place the glider on the track and the indexing block against the glider magnet and acquire this distance d_2, then, without moving the glider remove the block and acquire the distance d_3 to the glider flag.

The distance between the faces of the fixed and moving magnets is then $r = x - (d_3 - d_2) - d_1$ where x is the distance recorded by the rangefinder and approximately the distance between the centres of the two magnetic dipoles.

(!) With the elastic bumper located some 50 cm from the fixed magnet, turn on the air supply. Position the glider so that the magnets are 2-3 cm apart, then release the glider. The glider will oscillate

[1]The result is not exact, since close to C, where the magnetic force becomes important, the acceleration $\vec{a}$ is not proportional to $-\vec{v}$ anymore.

between A and C, repelled in turn by the elastic band and the repulsion of the magnets.

(!) Acquire some data and verify that the oscillations are well defined, without stray points, particularly at the extremes of the travel, then acquire around 120 seconds of data to record some 10–12 of these cycles. Note how the distance of closest approach ($\min(r)$) increases as the glider loses energy due to friction and losses in the elastic bumper.

(!) Use eXtrema to determine the maximum points at A and minimum points at C of the waveform, then for each $A - C - A$ cycle:

- fit some data on either side of C to get values for v_0 and v_1, then determine their average velocity v and the initial kinetic energy of the glider $K = 0.5mv^2$.
- calculate k, using Equation 5.11, with v_0, v_1, x_A and x_C, then with $r = r(C)$, use Equation 5.10 to obtain $U(r)$.

(!) Plot $U(r)$ as a function of r.

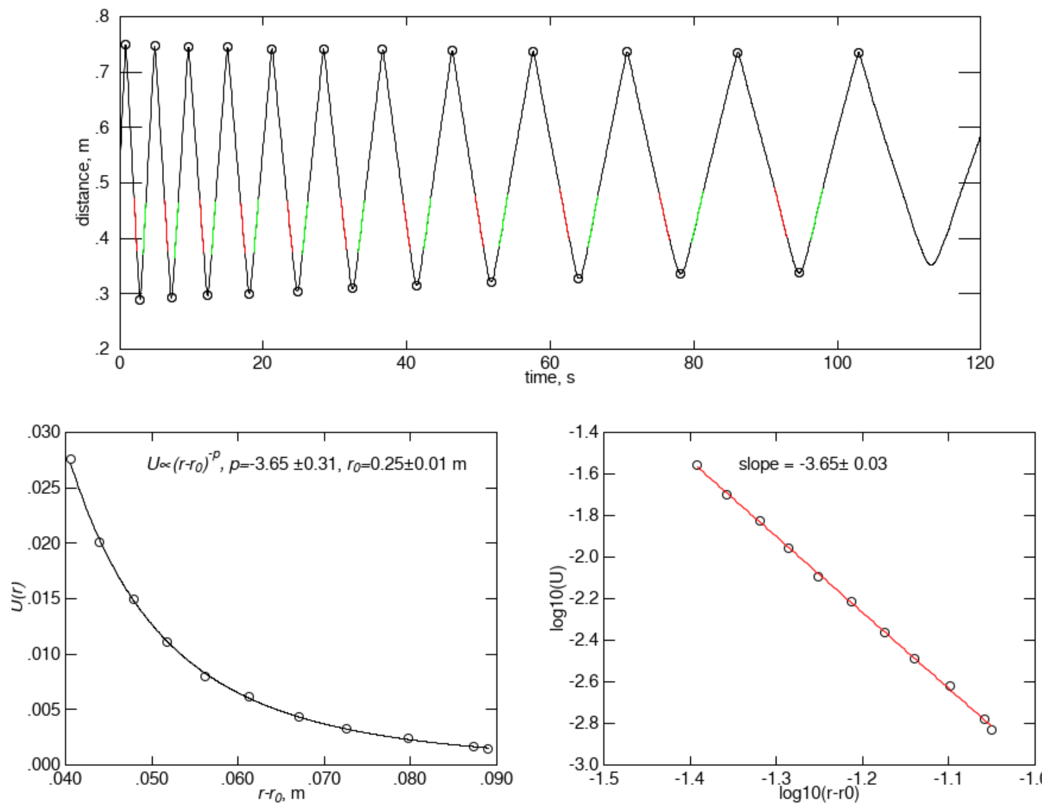

Figure 5.3: Top: Extrema analysis of glider magnet/bumper interactions on a flat track. Highlighted are the maximum points at A, minimum points x_C at C and fitted lines that yield v_0 and v_1. Bottom left: The potential energy data $U(r)$ fitted to yield p using the included macro code. Bottom right: $\log_{10}(U)$ *vs* $\log_{10}(r)$ fitted to a straight line.

(!) Explore a possible functional relation between $U(r)$ and r. For example, if $U(r)$ is linear in r, a plot on the linear scale along both axes will be a straight line. Alternatively, try plotting $U(r)$ vs. r on a log-log graph. A straight-line graph would correspond to $U(r) \propto r^n$, and by fitting you can determine the power n. Theoretically, for two point dipoles, the interaction potential is $U(r) \propto r^{-3}$.

Here is a short macro that fits $U(r) \propto (r - r_0)^p$ to estimate the power p for the interaction:

```
! select a measured point (r1,U1), ideally that of the nearest approach
! with U=a*(r-r0)^p and U1=a*(r1-r0)^p, then U=U1*((r-r0)/(r1-r0))^p
! change of variable: let b=r1-r0 => U(r)=U1*((r-r1)/b+1))^p
! then fit parameters are U1, b and p.

scalar\vary U1,b,p
U1=max(U)
r1=min(r)
b=0.5*r1
p=-log10(U1/min(U))/log10(max(r-b)/(r1-b))

fit\e2 log10(U)=log10(U1)+p*log10(1+(r-r1)/abs(b))
r0=r1-b
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(!) Try adjusting the range of fitted data for v_0 and v_1. How sensitive is the power p to these changes?

5.3 Optional: an alternate experimental procedure

An interesting alternative that you may wish to explore modifies the experimental approach slightly. Instead of using an elastic bumper on a level air track, it is possible to tilt the track up slightly at the far end, ensuring a slight increase in elevation as the glider rebounds and moves away from the stationary magnet. Since the increase in gravitational potential energy will be directly proportional to this elevation gain, and it in turn is related to the distance along the track, $\Delta h = r \sin\theta$, then $U_g(r) = mg\Delta h \propto \Delta h \propto r$. The resulting interaction potential is schematically illustrated in Fig. 5.4.

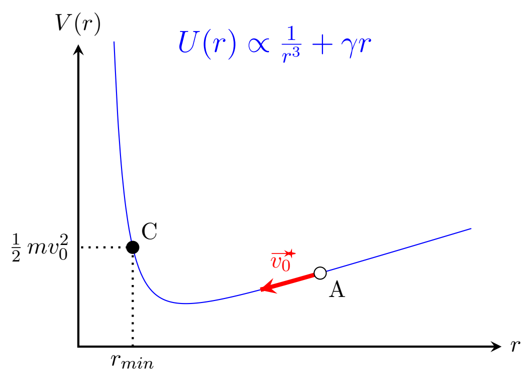

Figure 5.4: A modified interaction potential for a slightly tilted air track.

The change to the experimental procedure is minimal: start by releasing the glider at the far end of the raised track. Conservation of energy will ensure that after it bounces off the stationary magnet, it will not get any higher than this starting position on the track, so there is no danger of the glider flying off the end. With each successive bounce, the glider will be losing some energy to friction, and so at the next impact the kinetic energy of the glider entering the interaction region will be diminished, thus probing ever increasing values of r. The only modification to the analysis is to take into account the linear γr term in the potential. Its value can be estimated from the position of nearest approach at zero kinetic energy, which is the place where the glider will eventually settle after the bounces have stopped; it will, of course, depend on the tilt angle.

Can the exact form of $U(r)$ be calculated?

An approximate calculation of the force between two magnetic dipoles can be done using elliptic integrals for two cylindrical bar magnets aligned axially, if the distance between them is much greater than the magnet dimensions (radius, length)[2] but this is not the case in this experiment, so an empirical approach may be the best one can do.

[2] Vokoun, David; Beleggia, Marco; Heller, Ludek; Sittner, Petr (2009). "Magnetostatic interactions and forces between cylindrical permanent magnets". *J. Magnetism and Magn. Materials*, **321**(22):3758–3763. doi:10.1016/j.jmmm.2009.07.030.