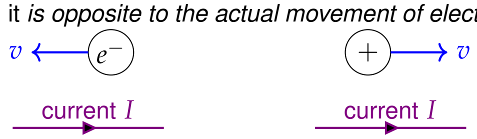
1 *Introductory concepts*

1.1 *Basic Physical Concepts*

In these introductory chapters, it will be very helpful to think of the electric current through the wires and other circuit elements as analogous to the water flow through a network of pipes. This is possible because the current in metals is carried by the movement of electronic charges. The electrons in the outer shells of metal atoms are not bound very strongly, and form an electronic gas of freely moving elementary charges, while the positively charged ions of the lattice remain in place. These *conduction electrons* move freely, very much like the water through pipes, and their movement creates the flow of charge, or current.

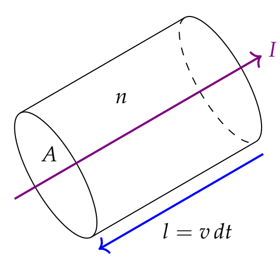
For historical reasons, the convention used in electric circuits is that of the so-called *conventional current* which is assumed to be carried by a flow of positive charges. This may well be true in currents through ionic liquids, where the moving charges are positive ions, but in metals the moving charges are negatively charged electrons, so when we draw the direction of an electric current in a wire, it *is opposite to the actual movement of electrons!*



TO SUMMARIZE, in metals, our view of the electric current is that of a 'gas' of free electrons moving in a crystal lattice, accelerated by the electric field of the applied voltage, impeded by collisions with the positive ions of the lattice.

There are an awful lot of these electrons: for example in copper (Cu) there are $n = 8.5 \times 10^{22}\, e/cm^3$, and as a result the electrons need to move very little to create a large flow of charge or a large current. Compare, for example, the average speed of thermal motion in an ideal gas at room temperature, or the order of $v_t \approx 10^5$m/s, and the drift velocity of the electronic gas that carries $I = 1$ A $= 1$ C/1 s of current through a typical wire of a 1 mm^2 cross-section:

$$
\begin{aligned}
\text{density of electrons:} \quad & n \\
e^- \text{ gas moves the distance:} \quad & l = v\,dt \\
\text{electrons passing by in time } dt: \quad & N = (A \times l) \times n = Anvdt \\
\text{current} \quad & I = \frac{dq}{dt} = \frac{eN}{dt} = eAnv \\
\text{average drift velocity:} \quad & v = \frac{I}{eAn}
\end{aligned}
$$



Verify the logic of the above calculation, determine the numerical value, then compare it to the average velocity v_t of thermal motions.

$v_d =$ __________ m/s, *vs.* $v_t \approx 10^5$ m/s

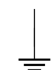
1.2 *Analogy with water flow*

Voltage is defined as the energy (work) involved in moving the charge, except it is defined as work *per unit charge*:

$$V \equiv \frac{W}{q}, \quad \text{in units of } 1\text{V} = \frac{1\text{J}}{1\text{C}}$$

As always, we capitalize the units of Coulombs, Amps, Volts, and Watts to pay our respects to the likes of Charles-Augustin de Coulomb (1736–1806), André-Marie Ampère (1775–1836), Alessandro Giuseppe Antonio Anastasio Volta (1745–1827), and James Watt (1736–1819), and the tradition will continue with other units we will encounter soon. The derivative units are denoted mA for milliamps, and μV for microvolts.

The definition of a voltage requires that you have a reference point: it matters from where the charge is coming. In circuits, this is the concept of a *ground*, and just like for gravitational potential energy, the point where you set the zero of the scale is arbitrary, and only changes *relative* to that point have physical meaning. (For example, the world uses *sea level* from which to measure heights.) Hence the other, perhaps more descriptive names that are used for voltage:



- electric potential (with respect to ground)
- potential difference (between two points)
- electromotive force (emf)

emf is of historical significance, but the use of the word 'force' can be confusing, so we will not use this one.

charge, C	mass, kg
current, $A = \text{C}\,\text{s}^{-1}$	mass flow, $\text{kg}\,\text{s}^{-1}$
voltage, $\text{V} = \text{J}\,\text{C}^{-1}$	P.E. per unit mass, $\text{J}\,\text{kg}^{-1}$

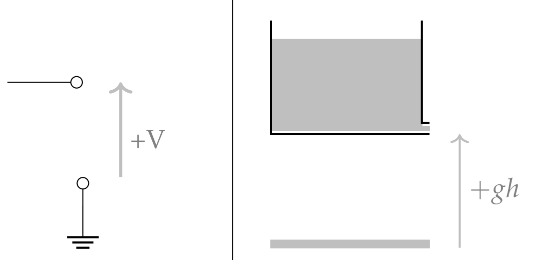
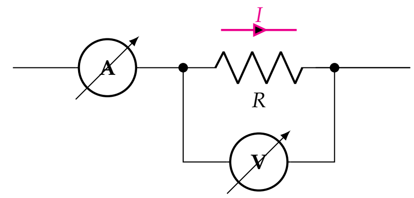


Table 1.1: Analogy between electric currents and the flow of water. A water tank elevated to height h provides the potential energy of gh per kg of water; voltage V is potential energy per Coulomb of charge.

1.3 *Resistance*

DH 1.4, S 1.3–1.4

The *drift velocity* of a charge in a conductor is limited by "friction-like" effects due to the interaction between the moving electronic charges and the stationary charges of the lattice, giving rise to *resistance*. In water flow, as a pipe narrows, resistance to flow increases. A perfect pipe is infinitely wide and offers no viscous "friction" where the water flow contacts the stationary pipe walls. Only superconducting wires offer a true zero resistance to electric flow.



OHM'S LAW, $V = IR$, relates the current *through* the resistance to the voltage *across* it. Note how the ammeter is connected in the same flow or "in series" with the resistor, and the voltmeter is connected across, or "in parallel" with the resistor, between the same two circuit *nodes*.

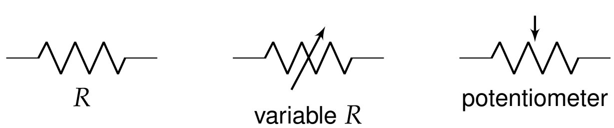
RESISTANCE IS A PROPERTY OF THE CONDUCTOR, expressed as $R = \rho L/A$ where ρ is the *resistivity* of the conducting material, L is the length and A is the cross-sectional area of the conductor. Intuitively, this relationship is as expected, since a smaller cross-sectional area or greater length increase resistance of a piece of wire. The shape-independent part is the resistivity ρ, and its value determines whether we consider the material a conductor (low ρ), an insulator (high ρ) or a superconductor (zero ρ). For an example, silver $\rho_{Ag} = 1.5 \times 10^{-8}\,\Omega$ m is a good conductor and carbon $\rho_C = 350 \times 10^{-8}\,\Omega$ m is a poor one.

Note that R is never really constant. In general, it could depend on the applied voltage, current, change in time, $R = R(V, I, t, ...)$, leading to non-linearity of VI-characteristic diagrams (for linear circuit elements, VI-characteristic is a straight line of constant slope of $1/R$). Since the resistivity is a function of temperature in most materials, so is the resistance; this dependence could be used to design electronic thermometers.

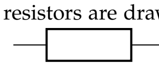
THE PHYSICAL RESISTORS that you encounter in the lab are tiny cylindrical objects, typically made out of a piece of resistive wire or a ceramic material, with two leads or contact pads at the ends. Before laser printing was available to deposit a fine label on the body of the resistor, a colour-coding scheme was developed and is still in use, each colour corresponding to a digit. The value of the resistance is determined by the combination of colour stripes. You will get a chance to learn this colour scheme in the lab, see Chapter 2.

DH 1.7

SCHEMATICALLY, the resistors on North American circuit diagrams look like a zigzag line:



In Europe, resistors are drawn like this:



In North America, this symbol is reserved for generalized *impedance*, see Chapter 7.

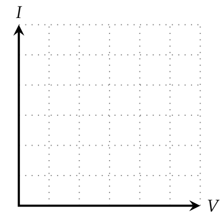
1.4 Meters

DISCUSS: What happens if an ideal 0–15 V DC voltmeter (measuring voltage) is connected across a 12 V car battery? What happens if the voltmeter is replaced with an ammeter (measuring current)?

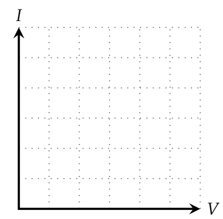
To answer this question, think about what an *ideal meter* is: it measures something without disturbing or changing in any way the underlying circuit. A voltmeter connects between two points in the circuit *in addition* to the other elements already connected, and it must not provide any additional pathway to the electric current. Hence its internal resistance must be as high as possible, infinitely high for an ideal voltmeter, for the current through this voltmeter to be exactly zero for any voltage it might be measuring.

To measure a current with an ammeter, on the other hand, requires that the meter be inserted directly into the flow of the current, so that the same current passes through the meter and the segment of the circuit we are measuring. It should not offer any additional resistance to the flow of charge, ideally its internal resistance must be zero. In this way, for any current passing through the meter, the voltage difference between its poles will be zero — as if it was not there, and the two points were connected by a perfectly conducting wire.

Now you are ready to return to the original questions. When you visit the lab, your workstation will contain a *multimeter*, a device that at a push of a button can switch from acting as a voltmeter (voltage) to an ammeter (current), or an ohmmeter (resistance). To prevent accidents that could destroy a meter when switching from voltage to current mode, a push of a button is not enough, one must also switch the connecting leads to a different socket on the multimeter!



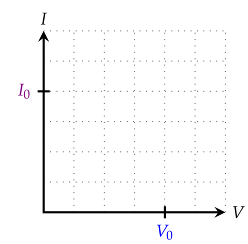
Draw a VI-characteristic of an ideal voltmeter.



Draw a VI-characteristic of an ideal ammeter.

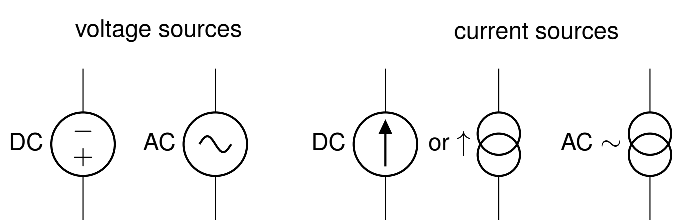
1.5 Sources

Sources provide constant voltage or current to the circuit. An ideal voltage source maintains a constant nominal voltage difference across its poles, no matter how high or low a current is being drawn from it by the attached circuit elements. An ideal current source maintains a constant nominal current through itself, no matter how high or low a voltage it needs to maintain across its poles to achieve that. Having considered above the ideal meters, you should have no difficulty sketching VI-characteristics of an ideal voltage source of nominal voltage V_0 and an ideal current source of nominal current I_0.



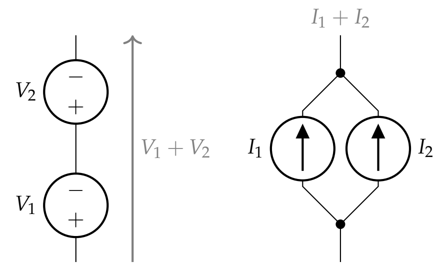
Draw VI-characteristics of an ideal voltage source of nominal voltage V_0 and an ideal current source of nominal current I_0.

Ideal sources on schematic diagrams are drawn like this:



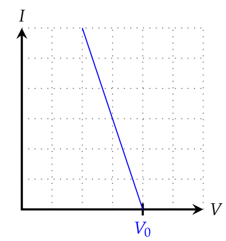
Note that ideal sources can only be added together in a particular way, namely voltage sources can only be added in series. Two ideal voltage sources, V_1 and V_2, added in parallel with each other would forever fight which voltage to maintain between the same two points in the circuit. Being ideal, there would be no compromise, and so this would only be possible if

$V_1 = V_2$, which is sort of pointless as the combination would be electrically equivalent to a single one of them. Similarly, two current sources, I_1 and I_2, can only be added in parallel, and not in series unless $I_1 = I_2$.

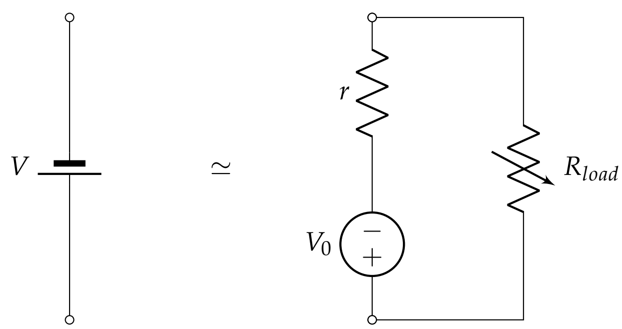


In contrast with the ideal sources, real sources typically remain linear over only a limited range of loads, so their VI-characteristics remain approximately straight lines, but the slopes are not strictly horizontal (for a current source, the output current is not independent of the voltage) or vertical (for a voltage source, the voltage typically decreases as the current drawn from the source increases).

DISCUSS: a real voltage source can be approximated by the following lumped circuit: an ideal voltage source V_0 in series with an internal resistance r. Verify that this circuit does have the correct VI-characteristic.



The VI-characteristics of an real voltage source shows a reduction in the output voltage as the demand from the load and, hence, the current increases. Check if this also describes the lumped circuit on the left.



DISCUSS: what would be a water-flow analog of a source? Clearly, it's not enough to just fill a tank once and lift it up: the water will flow, but only for a short time, and the flow will stop when the tank empties out. Remember that the sources like batteries are actually energy converters: they draw on their internal energy source, usually chemical in origin, to maintain the electric current flow. Make a sketch on the margins of a water-flow analog of an electric source.

1.6 Power

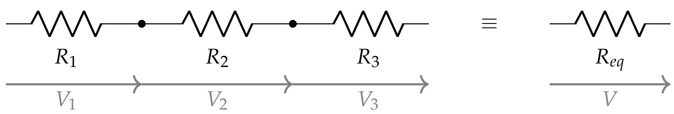
$$P = IV = I^2R = \frac{V^2}{R}$$

This is just a short reminder: ohmic devices — those that obey Ohm's Law — dissipate electric power by turning it into other forms of energy: heat, sound, light, *etc.* Their ability to do so is limited by their physical size, the heat-carrying capabilities of the surrounding medium, *etc.* The resistors you will encounter in your lab will typically be rated for 0.25 W, so keep that in mind and never exceed the rated power: the circuit may get hot to touch, causing burns to your fingers, or in the worst cases, becoming a fire hazard.

Note that one thing is common for both the ideal voltmeter and the ideal ammeter: they consume *no* power internally. *Real meters*, especially when measuring large currents, may dissipate a lot of power $P = I^2 r$ even if the internal resistance r is small. High currents can be a challenge to measure and may require special tricks.

1.7 *Circuit reduction*

IN-SERIES CONNECTION consists of a sequence of circuit elements, one after another:



Note how the current is the same through every component in the circuit: it's an expression of the conservation of charge, the entire charge flow that exits one circuit element enters the next one, as there is no alternative flow path.

Water-flow analogy here is: our pipes do not leak!

The voltages across each resistor add up in series, V_1 followed by V_2 followed byV_3 :

$$\begin{aligned} V &= V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = \\ &= I\left(R_1 + R_2 + R_3\right) = \\ &= R_{eq} \end{aligned} \tag{1.1}$$

where $R_{eq} = \sum_i R_i$ is the equivalent resistance for a group of in-series resistors, and the voltage drops V_i across each of the resistors R_i take a fraction of the total voltage V that is proportional to its resistance:

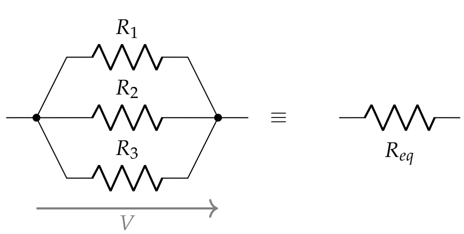
In-series resistances add up linearly!

$$V_i = IR_i = \frac{V}{\sum_i R_i} R_i = \frac{R_i}{\sum_i R_i} V. \tag{1.2}$$

Thus the in-series circuit is a *voltage divider*.

DH 1.9

IN-PARALLEL CONNECTION consists of a number of circuit elements, all connected between the same two circuit nodes:



Note how every component in the circuit sees the same voltage V across itself, but now the currents add up. Again, the conservation of charge is in play, the charge flow that enters a node divides up to exactly the sum of currents that exit the node. Again, there are no leaks — or extra sources of current — in the pipes!

$$\begin{aligned} I &= I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \\ &= V\frac{1}{R_{eq}}, \quad \text{with } \frac{1}{R_{eq}} = \sum_i \frac{1}{R_i} \end{aligned} \tag{1.3}$$

where $1/R_{eq} = \sum_i 1/R_i$ determines the equivalent resistance for a group of in-parallel resistors, and the currents I_i through each of the resistors R_i take a fraction of the total current I that is proportional to the inverse of its resistance:

In-parallel, inverses of resistances add up!

$$I_i = \frac{V}{R_i} = \frac{IR_{eq}}{R_i} = I\frac{1/R_i}{\sum_i 1/R_i}. \tag{1.4}$$

Thus the in-parallel circuit is a *current divider*. The current divides up proportionately to how easy it is to go through a particular path: the higher the resistance of the path, the smaller the fraction of the total current that goes through it.

The inverse of resistance is called *conductance*, $G \equiv 1/R$. Using that notation, $I_i = I\,(G_i/\sum_i G_i)$

The informal unit of G is "mho" — the reverse of "ohm", but its proper name is Siemens, 1 Siemens = $1\,\Omega^{-1}$

1.8 *Circuit reduction*

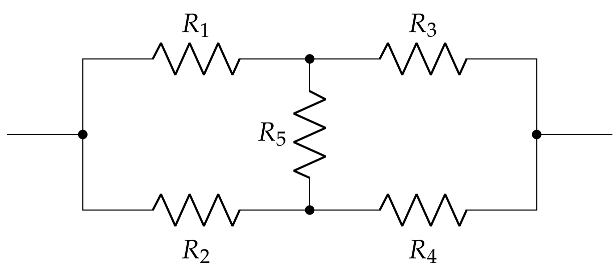
Complex combinations of resistors can be reduced by successive applications of the above two rules of equivalence. The principle is simple: identify a group of resistors that have only two leads going into the group (the "lasso" you throw around the group of resistors can only intersect the wires in two spots), and replace multiple resistors with their equivalent resistance. This is best illustrated with an example shown in Figure 1.1.

1.9 *What we haven't told you*

DISCUSS: what would be an appropriate lumped element representation of a *real* (*i.e.*, not ideal) voltmeter and ammeter? Use an ideal voltmeter/ammeter and an internal resistance r to draw a circuit that would represent a realistic device. As a further challenge, examine the multimeter you use in the lab; can you estimate its internal resistance when used as a voltmeter?

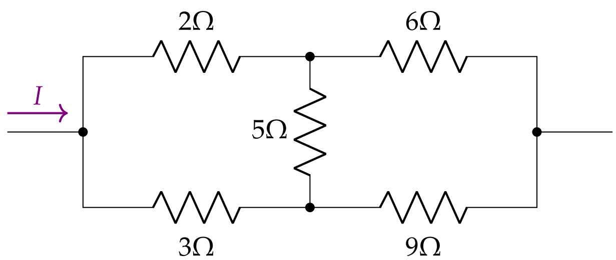
Hint: what is the largest resistance it can measure as an ohmmeter? Try a few.

DISCUSS: parallel/series circuit reductions are not always possible. Sometimes, there is simply no way to throw a "lasso" to have it cross a circuit in only two places, so that an equivalent resistance can be placed between these two points. Consider for example, this circuit:

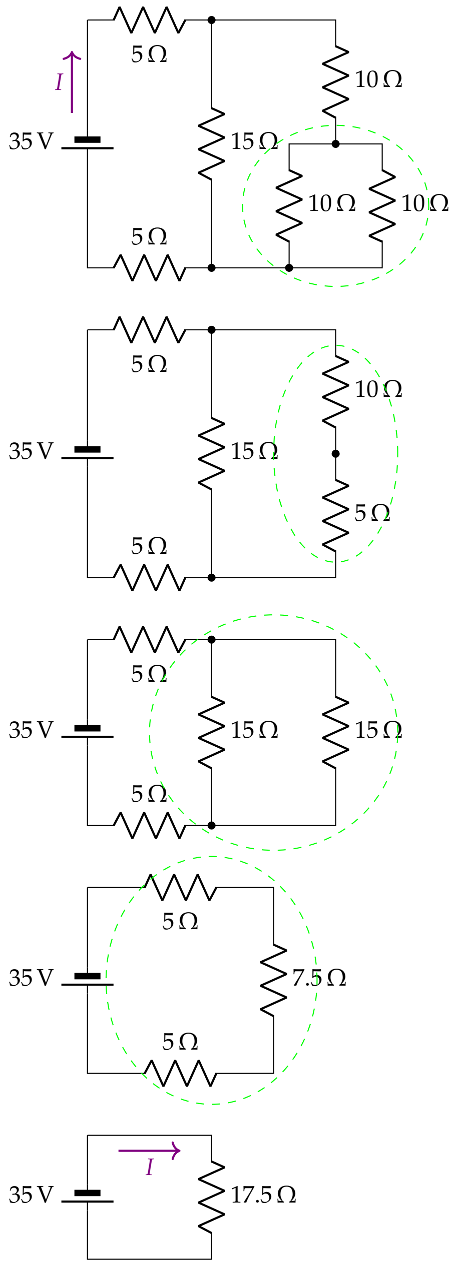


No matter how you try (do try!), every loop you draw intersects wires in *three* places. Clearly, we need some other way of dealing with more complex circuits, and we will get to it in the next Chapter.

EXCEPT . . . in some special cases one *can* reduce even such a circuit. There is a trick to it, and to see the solution you need to think about sending a current into this circuit and looking at the voltages that arise in various voltage dividers:



What is the voltage across the 5Ω resistor?



$$R_{eq} = \left(\frac{1}{10} + \frac{1}{10}\right)^{-1} = 5\Omega$$

$$R_{eq} = 10 + 5 = 15\Omega$$

$$R_{eq} = \left(\frac{1}{15} + \frac{1}{15}\right)^{-1} = 7.5\Omega$$

$$R_{eq} = 5 + 7.5 + 5 = 17.5\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{35V}{17.5\Omega} = 2\text{A}$$

Figure 1.1: Example of a circuit reduction by a series of successive replacements of in-series or in-parallel combinations with their equivalents.

As an exercise, reverse the direction, and calculate the currents in every resistor in all intermediate and, eventually, in the original circuit. Remember that every resistor in an in-series group shares the same current, and if two resistors in parallel are the same, the current will divide equally between them.