Experiment 2

Resistivity and Hall effect in a thin bismuth film

Thin films are an unusual system, and often their properties are different from those of the bulk materials. In the previous experiment, a thin film sample of Bi on the glass substrate was prepared. In this experiment we will measure the basic electrical characteristic of this sample, its resistivity, using the four-point van der Pauw technique. By placing the sample in an external magnetic field of known strength we will also be able to observe the Hall effect in this thin-film sample, and to estimate such fundamental quantities as density and mobility of free charge carriers.

2.1 Introduction

The conductivity σ of a solid depends on two parameters: n, the number of charge carriers per unit volume (in metals, the free electrons are the charge carriers, of charge $\bar{e}$ each), and the velocity with which the carriers move through the solid when an electric field is applied. The ratio of this drift velocity (in m/s) to the applied field (in V/m) is called *mobility* μ (in $m^2V^{-1}s^{-1}$). The two factors together define the *conductivity* of the material, as

$$\sigma = en\mu \tag{2.1}$$

The conductivity (or its inverse, the resistivity ρ) can in principle be determined by measuring the resistance of a block of length ℓ and cross-sectional area A $(= w \times d)$. If a current i is passed through the block, and a potential difference V is measured at two points ℓ apart along the direction of the current, then

$$\rho = \frac{V}{i} \times \frac{w \times d}{\ell}\,. \tag{2.2}$$

In practice one rarely has (or can make) a sample in this ideal shape, and a number of techniques are available to determine ρ from i and V measurements on samples of irregular shape. We will use a method developed by van der Pauw (1958), for isotropic ($\rho_{xx} = \rho_{yy} = \rho_{zz} = \rho$), homogeneous samples of constant thickness $d \ll \ell, w$, but otherwise of arbitrary shape.

The van der Pauw method

To do that, we will make use of four contact points, labelled A, B, C and D, on the sides of the flat sample as seen in Fig. 2.1; note that the method is restricted to the electrical contacts being at the periphery of the sample. A current i_{AB} is passed from A to B, and the resulting voltage difference V_{CD} is measured between contacts C and D; the ratio V_{CD}/i_{AB} has the dimension of "resistance". By *reciprocity theorem* (Irwin, 1987), the same result can also be obtained as the ratio V_{AB}/i_{CD}, and the resulting resistance is, therefore

$$R_1 = R_{AB,CD} = \frac{V_{CD}}{i_{AB}} = R_{CD,AB} = \frac{V_{AB}}{i_{CD}}\,. \tag{2.3}$$

To compensate for voltage bias due to contact effects, one could also reverse the direction of the current and hence, the polarity of the measured voltage, thus producing a total of four possible measurements of R_1. The best precision is obtained by taking the average of all four values.

Next, one can pass the current i_{BC} from B to C and measure the voltage V_{DA} or, by reciprocity, V_{BC} and i_{DA}, to determine

$$R_2 = R_{BC,DA} = \frac{V_{DA}}{i_{BC}} = R_{DA,BC} = \frac{V_{BC}}{i_{DA}}\,. \tag{2.4}$$

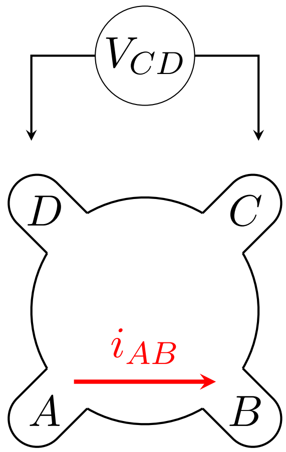

Figure 2.1: A schematic illustration of the four contact points.

According to van der Pauw (1958), R_1 and R_2 are related to the sample resistivity ρ and thickness d by:

$$e^{-\pi d R_1/\rho} + e^{-\pi d R_2/\rho} = 1 \tag{2.5}$$

Equation 2.5 does not have a closed-form solution for ρ, but it can be rewritten as:

$$\rho = \frac{\pi d}{\ln 2} \cdot \frac{(R_1 + R_2)}{2} f\left(\frac{R_1}{R_2}\right) \tag{2.6}$$

where the so-called van der Pauw function $f(x)$ of the argument $x = R_1/R_2$ satisfies

$$\cosh\left[\frac{\ln 2}{f(x)}\frac{x-1}{x+1}\right] = \frac{1}{2}\exp\left[\frac{\ln 2}{f(x)}\right] \tag{2.7}$$

Values of $f(x)$ can be calculated numerically (see Julury, 2012, for matlab/octave code) or looked up in a table; for small $x = R_1/R_2$, an approximate expression $f(x) \approx 1/\cosh(\ln(x)/2.403)$ may be used, with an error of less than 0.1% for $x < 2.2$ and less than 1% for $x < 4.3$. After measuring R_1 and R_2 and calculating R_1/R_2 for the sample, its resistivity ρ can be calculated from Equation 2.6. Notice that the van der Pauw method requires the measurement of only the thickness d of the sample; the standard method involving a block of material requires three separate measurements of the sample dimensions.

The Hall effect

The Hall effect is the sideways deflection of the moving charge carriers in the sample due to a magnetic field $\vec{B}$ perpendicular to the current $\vec{i}$. An electron travelling with velocity $\vec{v}$ in the (negative) $-x$-direction (*i.e.*, a positive x-direction of the conventional current) experiences a Lorentz force $q\vec{v} \times \vec{B}$, with $q = \overline{e}$, in the (positive) z-direction due to a $\vec{B}$ field in the (positive) y-direction; the result is a potential difference, the Hall voltage V_H, that develops between the sides of the sample in the z-direction. Placing the thin-film sample in an external magnetic field, with the normal to the glass slide (the y-axis) along the field, and sending a current between the two opposite contact points (i_{AC}, as in Fig. 2.2, or i_{BD}) creates a voltage ΔV_H between the other two contact points (V_{BD} and V_{AC}, respectively).

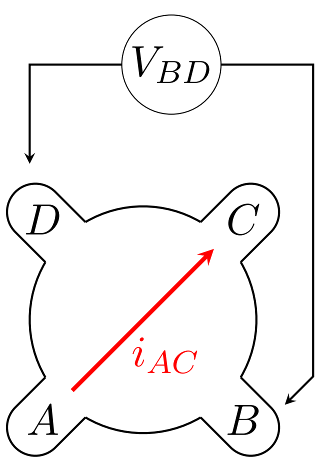

Figure 2.2: A schematic illustration of the four contact points.

This voltage is the measure of the electrical field $\vec{E}$ created by the sideways displacement of the electrons, and the force due to this field balances out the Lorentz force: $evB = eE$ (magnitudes only). In a "block" of material of dimensions $\ell \times A = \ell \times w \times d$, and charge carrier density n, the current i is the total charge per unit time t:

$$i = \frac{(\ell \times A)ne}{t} = vAne = v(w \times d)ne\,, \tag{2.8}$$

and this provides an expression for v in term of i, so the Hall voltage can be written as

$$\Delta V_H = wE = wvB = \frac{iB}{dne} \,. \tag{2.9}$$

The Hall coefficient R_H, defined as

$$R_H = \frac{\Delta V_H \, d}{iB} \,, \tag{2.10}$$

is directly related to the the carrier concentration n and the charge of each carrier, e, as

$$n = \frac{1}{eR_H} \tag{2.11}$$

Once n is known, the mobility μ of the charge carriers can be calculated (from Eq. 2.1) as

$$\mu = \frac{\sigma}{en} = \frac{1}{\rho en} = \frac{R_H}{\rho} \,. \tag{2.12}$$

2.2 Measurement checklists

Resistivity measurement

To determine ρ for a thin-film bismuth film:

1. Place the film in the sample holder and attach the four contacts A, B, C and D to the film. This is the most delicate part of the lab, as the thin layer of metal on the glass substrate is easily scratched, disrupting the continuity of the film, or even breaking the electrical contact. The best way to obtain a good contact is to:

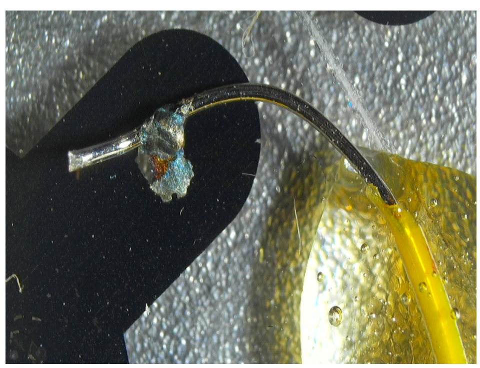

Figure 2.3: Making a good contact: note a small drop of silver paint, the epoxy holding the wire securely, and a slightly upturned tip of the wire.

   (a) use four thin (wire-wrap) wires, stripped of insulation for about one cm at the end, and slightly bent up so that the sharp tip of the wire does not scratch the sample surface;

   (b) using masking tape, tape down the glass slide by its edges; temporarily holding the wires in place with thin strips of the tape, arrange the wires to make clean contact with the four pads, and place a drop of quick-setting epoxy to fix the wires in place;

   (c) using a sharp tool (a sharpened tip of a handle of a cotton swab works well), gently place a drop of conductive silver paint onto the place where the wire touches the thin film, and let it dry; use small amounts of paint, as it shrinks a little while drying, and may crack the thin film disrupting the electrical continuity;

(d) use an ohmmeter to verify that all four wires make electrical contact with the sample; if needed, repeat the previous step, spreading the silver paint until the contact is good; be careful not to short nearby wires.

An alternative method is to solder the wires into place using indium as solder. The process is delicate and requires precise temperature control of the tip of the soldering iron, so that it is just hot enough to melt In, but not so hot as to melt the Bi film underneath it. If you have some experience in soldering, this may prove to be a superior technique, but complete novices usually find the silver-paint method simpler. Note that in both cases it is necessary to provide good strain relief by placing a drop of epoxy to hold wires in place on the glass slide.

2. Measure i_{AB} and V_{CD} using a current source, an ammmeter in series, and a voltmeter, and calculate R_1. Reverse the direction of the current leads and determine R_1 again. The two values of R_1 should be identical unless you have poor point contacts that act as tiny diodes. The best results are obtained by systematically varying the current and fitting a straight line to the resulting data set that includes all of the data, for both current directions. Repeat for i_{CD} and V_{AB}.

3. Similarly determine $R_2 = I_{BC}/V_{AD}$; also check for $R_2 = i_{AD}/V_{BC}$. Calculate average values of R_1, R_2 and R_1/R_2.

4. Determine the appropriate value for the van der Pauw function $f(R_1/R_2)$[1] for your sample and calculate ρ.

Hall-effect measurement

To determine n and μ for a thin-film bismuth film:

1. To observe the Hall effect, apply a current i_{AC} and measure the voltage V_{BD}. Place the sample between the poles of the electromagnet (you can use masking tape to tape the sample to the center of the pole of the magnet), which puts the magnetic field $\vec{B}$ along the normal to the sample. Measure i_{AC} and V_{BD} for several values of $\vec{B}$; the measured voltage changes to a new value $V_{BD} + \Delta V_H$.

   This can be repeated for a range of currents i_{AC}. Reverse the direction of the current to check for non-Ohmic (diode-like rectifying) contacts. Swapping the electromagnet leads at the power supply (but making sure to turn the magnet current down to zero first!) allows for reversing the direction of the magnetic field, and of the sign of ΔV_H to compensate for thermoelectric effects at contact points or other sources of voltage bias.

2. Similar values can be obtained by applying the current i_{BD} and measuring the resulting voltage V_{AC}. Reconnect the leads to measure i_{BD}, and V_{AC}, and measure ΔV_H again. Again, reverse the direction of the current i_{BD}, and/or reverse the direction of the $\vec{B}$-field as well as vary its strength to change the sign of ΔV_H.

3. The Hall coefficient R_H is related to ΔV_H by:

$$\Delta V_H = R_H \frac{i_{AC} B}{d} . \tag{2.13}$$

   Calculate R_H using the average of ΔV_H values obtained in the previous two steps, or by performing a fit to $\Delta V_H = \Delta V_H(i_{AC}, B)$ using R_H as a parameter of the fit, and calculate n and μ for Bi, using Eqs. 2.11–2.12. Compare n (the number of "free" electrons per unit volume) to the number of Bi atoms per unit volume.

Be sure to include the tables with raw V and i values in an Appendix of your lab report.

[1] Pre-calculated values can be downloaded from `https://juluribk.com/images/van_der_pauw_correction_factor.csv` and plotted or interpolated numerically.

References

1. D. Halliday and R. Resnick, *Fundamentals of Physics*, Chapters 28, 30-4.

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4. Van der Pauw, L.J. A method of measuring specific resistivity and Hall effect of discs of arbitrary shape. *Philips Research Reports*, **13**:1-9, 1958.
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6. J. D. Irwin, *Basic Engineering Circuit Analysis*, 2nd ed. Macmillan, New York, 1987. Chapter 15.