Gauge Transformations

1. (a) Consider a system where

$$V = 0; \qquad \vec{A} = \frac{Ct}{2\pi\epsilon_o}\frac{1}{s}\hat{s}$$

Determine $\vec{E}, \vec{B}, \rho$ and $\vec{J}$ using cylindrical co-ordinates. *Hint:* Review Griffiths example 2.1.

(b) What are the units of C ?

(c) Consider the same system under a gauge transformation where

$$\lambda = -\frac{Ct}{2\pi\epsilon_o}\ln(\frac{s}{s_o})$$

Determine the new $\vec{A}'$ and V'.

(d) Notice that we have produced a situation where $V = 0$ is set a distance s_o from the axis of cylindrical symmetry. Compare V' with a direct calculation of V using superposition: i.e.

$$V = \sum \frac{q_i}{4\pi\epsilon_o}\frac{1}{|\vec{r} - \vec{r_i}|} = \int \frac{dq}{4\pi\epsilon_o}\frac{1}{|\vec{r} - \vec{r_i}|}$$

2. Consider a system where

$$V = 0; \qquad \vec{A} = \frac{\mu_o I}{2\pi}\ln(\frac{s_o}{s})\hat{z}$$

Determine $\vec{E}, \vec{B}, \rho$ and $\vec{J}$ using cylindrical co-ordinates.

3. Griffiths problem 10.4