Wave Equation

1. (a) By substitution, show that $y = f(x - vt) + g(x + vt)$ is a solution to the one-dimensional wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

   (b) Which of the following functions would be acceptable solutions of the one-dimensional wave equation:

      i. $y = A(x - vt)^3$
      ii. $y = A(x^3 - vt)^3$
      iii. $y = A\sin(kx - \omega t)$
      iv. $y = A\sin(kx - \omega t^2)$

2. (a) Using Maple or some other program and setting $A = 1, v = 2$, make plots of the following functions

      i. $y = A\exp[-(x - vt)^2]$ for $t = 0, 1, 2, 3$. Put all four curves on the same plot.
      ii. $y = A\exp[-(x + vt)^2]$ or $t = 0, 1, 2, 3$. Put all four curves on the same plot.

   (b) What do you conclude about the sign in the argument of the function $(x \pm vt)$?

3. By substitution, show that $f(\vec{r}, t) = A\exp[i(\vec{k}\cdot\vec{r} - \omega t + \delta)]$ is a solution of the three-dimensional scalar wave equation:

$$\nabla^2 f = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$$

   Do the calculation in cartesian co-ordinates and note that $k = |\vec{k}|$ and $\frac{\omega}{k} = v$ and δ is just a constant phase shift to allow matching to initial conditions.