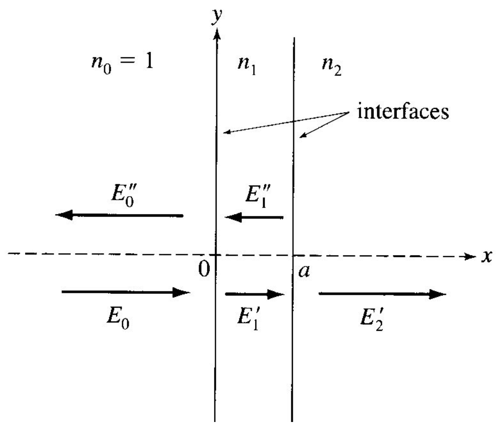
Anti-reflection coatings

1. An important application of wave optics is to design a lens-air interface that transmits 100% of the light incident on it. As you will show in this problem, this can be achieved by coating the lens with a layer of dielectric having just the right thickness and index of refraction. The figure below shows the geometry. The directions of propagation of the plane waves are indicated by the arrows. The electric vectors are all taken to point in the $+\hat{y}$ direction.



   (a) Write out the $\vec{E}$ and $\vec{B}$ in each of the three regions $x \leq 0$, $0 \leq x \leq a$ and $x \geq a$

   (b) Apply the boundary conditions at the interface $x = 0$ between the air and the dielectric and obtain two equations in E_o, E_o'', E_1' and E_1''. Remember that the magnitudes of $\vec{E}$ and $\vec{B}$ are related.

   (c) Apply the boundary conditions at the interface $x = a$ between the air and the glass and obtain two equations in E_1', E_1'' and E_2'.

   (d) We seek now a solution to the boundary conditions in which there is no reflected wave, (i.e. one where $E_o'' = 0$). Set $E_o'' = 0$ in the two equations you determined in (b) and then solve for E_1' and E_1'' in terms of E_o.

   (e) Then substitute for E_1' and E_1'' in the two equations you determined in (c) and solve for the ratio of the equations. E_o and E_2'

will cancel out. After some simplification the result is

$$\frac{n_2}{n_1} = \frac{n_1 i \sin k_1 a + \cos k_1 a}{n_1 \cos k_1 a + i \sin k_1 a}$$

(f) Now the right hand side of the above equation must be real. Show that the thinnest non-reflective coating has $a = \frac{\lambda_1}{4}$ where $n_1 = \sqrt{n_2}$.