Magnetic and Electric Dipole Radiation

1. Beginning with the vector potential for a neutral loop of radius b with center at the origin and lying in the x-y plane which carries current $I = I_o \cos \omega t$, show that

$$\vec{E}_{rad} \sim -\frac{\mu_o m_o \omega^2}{4\pi c} \frac{\cos\left[\omega(t - r/c)\right]}{r} \sin(\theta)\hat{\phi}$$
$$\vec{B}_{rad} \sim -\frac{\mu_o p_o \omega^2}{4\pi c^2} \frac{\cos\left[\omega(t - r/c)\right]}{r} \sin(\theta)\hat{\theta}$$
$$\vec{S}_{rad} \sim \frac{\mu_o m_o^2 \omega^4}{16\pi^2 c^3} \frac{\cos^2\left[\omega(t - r/c)\right]}{r^2} \sin^2(\theta)\hat{r}$$

2. (a) Show that the units of $\sqrt{\frac{\mu_o}{\epsilon_o}}$ are Ω.

   (b) Calculate the value of $\sqrt{\frac{\mu_o}{\epsilon_o}}$ which is called "the impedance of free space".

3. A Hertzian dipole is one where ($\vec{p} = p_o \cos \omega t \hat{z}$). Show that the "antenna resistance" of such a dipole , which a measure of how efficient a radiator it is, is given by

$$R_{ant} = \frac{P_{avg}}{I_{rms}^2} = (20\Omega)\,(kd)^2$$

   Our approximation assumes $d << \lambda << r$. Hence $kd << 1$.

4. (a) Calculate R_{ant} as some numerical factor times $(kb)^n$ for magnetic dipole radiation. Determine n.

   (b) Calculate R_{ant} (and thus the efficiency) of both point electric and magnetic dipole radiators when $kd \approx kb \approx 0.1$. Our calculations for both electric and magnetic dipole radiation assumed λ much greater than the dimensions of the source. Neither the point electric dipole, nor point magnetic dipole is a very efficient radiator in contrast to a half-wave antenna where $R_{ant} = 73\Omega$.