Dielectric Properties of Materials

1. To determine a vector field, one needs to know BOTH the divergence and curl and a set of boundary conditions. This is the Helmholtz theorem.

   (a) Explain why for electrostatic conditions it is sufficient to know the divergence of $\vec{E}$ to determine $\vec{E}$ using Gauss's theorem.

   (b) Explain under what conditions one can use the divergence theorem to determine $\vec{D}$ from $\vec{\nabla} \cdot \vec{D} = \rho_f$.

2. A dielectric object that has a quasi-permanent polarization when the applied field is 0 is called an *electret*. Consider a uniformly polarized electret in the shape of a cylinder of height h and radius $10h$. The polarization of the electret is $P\hat{z}$, where $\hat{z}$ is parallel to the cylinder axis.

   (a) Sketch the electric field lines.

   (b) Calculate the electric field $\vec{E}$ and $\vec{D}$ at the center of the cylinder. Because the radius is large compared to the height, you may neglect edge effects.

   (c) Caculate the electric field $\vec{E}$ and $\vec{D}$ on the midplane of the cylinder, at a distance $100h$ from the cernter. Because the distance is large compared to the radius, the dipole dominates the multipole expansion.

   (d) There is no free charge in this problem. What is producing $\vec{D}$ in part (c)?

3. A point charge Q is embedded in a l.i.h. dielectric sphere of susceptibility χ_e and radius R. Q is a free charge.

   (a) What is the bound charge $\rho_b(r)$ and $\sigma_b(r)$ in and on the sphere? How does the results vary with R?

   (b) What is the total charge enclosed by the sphere?