Review: Maxwell's Equations

1. Consider a cylindrical wire carrying a steady current density $\vec{J}$. Recall the connection between current density and charge density: $\vec{J} = \rho\vec{v}$. In our wire, negatively charged electrons of charge density ρ_- move with average velocity $\vec{v}$ and carry the current while the positive ion cores - of charge density ρ_+ - are fixed in place and do not contribute to the current in the lab frame. Recall that, due to the Lorentz force, parallel currents attract each other. Thus you may wonder whether all of the free electrons should then be forced to the center of the wire. This does not happen because the aforementioned magnetic force is balanced by the electrical force due to the electric field due to the excess negative charge.

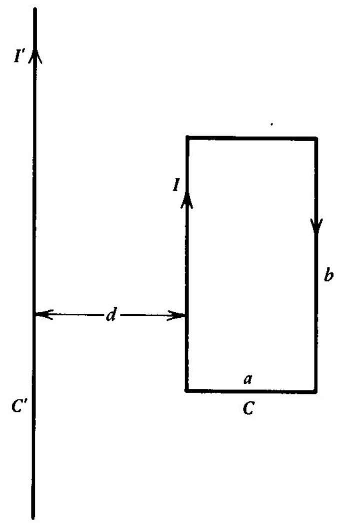
   The end effect is that while the wire is electrically neutral on a macroscopic scale, it has a microscopically inhomogeneous charge density: The surface of the wire is positively charged while the interior of the wire $\rho_- > \rho_+$ and has a net negative charge density.

   (a) Show that
   $$\rho_- = -\gamma^2\rho_+$$
   where
   $$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$
   .

   (b) Determine the positive surface charge density σ_+

   Note that the effect is very small because $v << c$ for a typical situation.

2. Consider the circuits shown in the figure below. Suppose the current in the infinitely long straight circuit C' is given by $I = I_o e^{-\lambda t}$ where I_o and λ are constants. Find the induced emf that will be produced in the rectangular circuit of this same figure. What is the direction of the induced current?


I'
I
b
d
a
C'
C