Gauge Transformations

1. Which of the following potentials are in the Coulomb Gauge and which are in the Lorentz Gauge? Note that they are not mutually exclusive.

   (a) $V = 0$; $\quad \vec{A} = -\frac{\mu_o k}{4c}(ct - |x|)^2 \hat{z}$

   (b) $V = 0$; $\quad \vec{A} = -\frac{1}{4\pi\epsilon_o}\frac{qt}{r^2}\hat{r}$

   (c) $V = 0$; $\quad \vec{A} = A_0 \sin(kx - \omega t)\hat{y}$ (Use cartesian co-ordinates; A_o, ω, k are constant.)

2. Later on in the course, we will study electric dipole radiation. An oscillating electric dipole will produce electromagnetic waves. The equations below, which have azimuthal (ϕ) symmetry in spherical co-ordinates, are the potentials associated with an oscillating electric dipole:

$$V(r, \theta, t) = -\frac{p_o \omega}{4\pi\epsilon_o c}(\frac{\cos\theta}{r}) \sin[\omega(t - r/c)]$$

$$\vec{A}(r, \theta, t) = -\frac{\mu_o p_o \omega}{4\pi r} \sin[\omega(t - r/c)]\hat{z}$$

   p_o, which is the magnitude of the oscillating dipole, and $\omega = 2\pi f$, which is the angular frequency of the radiation, are constant. As usual, $c = \frac{1}{\sqrt{\epsilon_o \mu_o}}$ is the speed of light. Show that in the limit $r >> \frac{c}{\omega} = \frac{\lambda}{2\pi}$ where r is the distance from the dipole and λ is the wavelength of the radiation, that these potentials satisfy the Lorentz Gauge condition.