PHYS 6P12 PROF. HARROUN

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① MAXWELL'S EQUATIONS ARE A SET OF COUPLED PARTIAL DIFFERENTIAL EQUATIONS, THAT, TOGETHER WITH THE LORENTZ FORCE LAW, FORM THE FOUNDATION OF CLASSICAL ELECTROMAGNETISM CLASSICAL OPTICS, AND ELECTRIC CIRCUITS.

IN DIFFERENTIAL FORM THEY ARE:

GAUSS' LAW $\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$

GAUSS' LAW FOR MAGNETISM $\nabla \cdot \vec{B} = 0$

MAXWELL-FARADAY EQUATION $\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$

AMPÉRE'S LAW $\nabla \times \vec{B} = \mu_0 \left( \vec{J} + \varepsilon_0 \frac{d\vec{E}}{dt} \right)$

TAKING THE CURL OF THE CURL EQUATIONS: WE ARRIVE AT

$$\frac{1}{c^2}\frac{d^2\vec{E}}{dt^2} - \nabla^2\vec{E} = 0 \qquad c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

$$\frac{1}{c^2}\frac{d^2\vec{B}}{dt^2} - \nabla^2\vec{B} = 0$$

(2) THE SCHRÖDINGER EQUATION IS A LINEAR PARTIAL DIFFERENTIAL EQUATION THAT DESCRIBES THE STATE FUNCTION OF A QUANTUM MECHANICAL SYSTEM

$$i\hbar \frac{d}{dt} \Psi(\vec{r},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + V(\vec{r},t)\right]\Psi(\vec{r},t)$$

$$\hat{H}|\Psi\rangle = E|\Psi\rangle$$

$$\sigma(x)\,\sigma(p_x) \geq \frac{\hbar}{2}$$

$$\hat{p} = -i\hbar\frac{d}{dx} \Rightarrow \hat{H} = \frac{\hat{p}^2}{2m} + V(x)$$

LET $V = 0 \Rightarrow -E\psi = \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}$

SOLUTIONS ARE PLANE WAVES:

$$\left.\begin{array}{l} \psi_E(x) = A e^{ikx} + B e^{-ikx} \\ \text{WHERE } k = \sqrt{\frac{2mE}{\hbar^2}} \end{array}\right\} \text{ IF } E > 0$$

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FOR SIMPLE HARMONIC OSCILLATOR

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

(3) THE FRACTION OF PARTICLES WITHIN VELOCITY VOLUME $d^3v$ AT VELOCITY $v$ IS

$$f(v)\,d^3v = \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}}\,d^3v$$

$$\int_0^\infty f(v)\,d^3v = 1$$

THE MOST PROBABLE SPEED $v_p$ IS

$$\frac{df(v)}{dv} = -8\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v\,e^{-mv^2/2kT}\left(\frac{mv^2}{2kT}-1\right) = 0$$

$$\Rightarrow \frac{mv_p^2}{2kT} = 1 \Rightarrow \boxed{v_p = \sqrt{\frac{2kT}{m}}} = \sqrt{\frac{2RT}{M}}$$

WHERE $m$ IS PARTICLE MASS AND $M$ IS THE MOLAR MASS

THE MEAN SPEED IS (SETTING $b = \frac{m}{2kT}$)

$$\langle v\rangle = \int v\,f(v)\,dv = 4\pi\left(\frac{b}{\pi}\right)^{3/2}\int_0^\infty v^3 e^{-bv^2}\,dv$$

$$= 4\pi\left(\frac{b}{\pi}\right)^{3/2}\frac{1}{2b^2} = \sqrt{\frac{4}{\pi b}}$$

$$\boxed{\langle v\rangle = \sqrt{\frac{8kT}{\pi m}}} = \frac{2}{\sqrt{\pi}}v_p$$