

① MAXWELL'S EQUATIONS ARE A SET OF COUPLED PARTIAL DIFFERENTIAL EQUATIONS, THAT, TOGETHER WITH THE LORENTZ FORCE LAW, FORM THE FOUNDATION OF CLASSICAL ELECTRO MAGNETISM, CLASSICAL OPTICS, AND ELECTRIC CIRCUITS.

IN DIFFERENTIAL FORM THEY ARE:

GAUSS' LAW $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$

GAUSS' LAW FOR MAGNETISM $\nabla \cdot \vec{B} = 0$

MAXWELL-FARADAY EQUATION $\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$

AMPÈRE'S LAW $\nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{d\vec{E}}{dt} \right)$

TAKING THE CURL OF THE CURL EQUATIONS:
WE ARRIVE AT

$$\frac{1}{c^2} \frac{d^2 \vec{E}}{dt^2} - \nabla^2 \vec{E} = 0$$

$$\frac{1}{c^2} \frac{d^2 \vec{B}}{dt^2} - \nabla^2 \vec{B} = 0$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

- ② THE SCHRÖDINGER EQUATION IS A LINEAR PARTIAL DIFFERENTIAL EQUATION THAT DESCRIBES THE STATE FUNCTION OF A QUANTUM MECHANICAL SYSTEM.

$$i\hbar \frac{\partial}{\partial t} \Psi(\vec{r}, t) = \left[-\frac{\hbar^2}{2m} \nabla^2 + V(\vec{r}, t) \right] \Psi(\vec{r}, t)$$

$$\hat{H} |\Psi\rangle = E |\Psi\rangle$$

$$\sigma(x) \sigma(p_x) \geq \frac{\hbar}{2}$$

$$\hat{p} = -i\hbar \frac{d}{dx} \Rightarrow \hat{H} = \frac{\hat{p}^2}{2m} + V(x)$$

$$\text{LET } V=0 \Rightarrow -E\psi = \frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2}$$

SOLUTIONS ARE PLANE WAVES:

$$\psi_E(x) = A e^{ikx} + B e^{-ikx} \quad \left. \vphantom{\psi_E(x)} \right\} \text{ IF } E > 0$$

$$\text{WHERE } k = \sqrt{\frac{2mE}{\hbar^2}}$$

FOR SIMPLE HARMONIC OSCILLATOR

$$E_n = \left(n + \frac{1}{2}\right) \hbar \omega \quad \psi_n(x) = \frac{1}{\sqrt{n!}} \left(\sqrt{\frac{m\omega}{2\hbar}}\right)^n \left(x - \frac{\hbar}{m\omega}\right) \frac{d}{dx}$$

③ THE FRACTION OF PARTICLES WITHIN VOLUME d^3 AT VELOCITY v IS

$$f(v) d^3v = \left(\frac{m}{2\pi kT} \right)^{3/2} e^{-\frac{mv^2}{2kT}} d^3v$$

$$\int_0^\infty f(v) d^3v = 1$$

THE MOST PROBABLE SPEED v_p IS

$$\frac{df(v)}{dv} = -8\pi \left(\frac{m}{2\pi kT} \right)^{3/2} v e^{-mv^2/2kT} \left(\frac{mv^2}{2kT} - 1 \right) = 0$$

$$\Rightarrow \frac{mv_p^2}{2kT} = 1 \Rightarrow v_p = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}}$$

WHERE m IS PARTICLE MASS AND M IS THE MOLAR MASS

THE MEAN SPEED IS (SETTING $b = \frac{m}{2kT}$)

$$\langle v \rangle = \int v f(v) dv = 4\pi \left(\frac{b}{\pi} \right)^{3/2} \int_0^\infty v^3 e^{-bv^2} dv$$

$$= 4\pi \left(\frac{b}{\pi} \right)^{3/2} \frac{1}{2b^2} = \sqrt{\frac{4}{\pi b}}$$

$$\langle v \rangle = \sqrt{\frac{8kT}{\pi m}} = \frac{2}{\sqrt{\pi}} v_p$$