Experiment 5

Viscosity and drag

Drag force arises when an object moves through a fluid or, equivalently, when fluid flows past an object. In general, the drag force grows larger with increased flow velocity, but viscosity is a complex phenomenon that cannot be reduced to the simple relationship "drag force is proportional to velocity".

The origin of the drag force F_d lies in the need to displace the particles of the fluid out of the way of a moving object. At low velocities the movement of the fluid is smooth (*laminar*) and the faster the object moves, the greater is the amount of fluid that has to get out of the way, giving rise to the linear relationship

$$F_d^{(viscous)} = a\eta v$$

where v is the velocity of the object relative to the fluid, η is the *coefficient of viscosity*, and a is the "size" of the object; for a sphere of radius r, $a = 6\pi r$. The larger the size, the greater is the amount of fluid that needs to get out of the way, hence it is not surprising that the relationship is linear with respect to a as well.

The story gets far more complicated for higher velocities, unusual object shapes, of unusual fluids. At higher velocities or around oddly-shaped objects the flow stops being smooth and turns *turbulent*, which requires more energy and causes the drag force to switch to the quadratic regime, where $F_d \propto v^2$,

$$F_d^{(inertial)} = S\frac{\rho_0 v^2}{2} \ .$$

Note that this expression represents an inertial rather than a viscous force, and instead of the the viscosity, η, the fluid density, ρ_0, enters the formula. S is the cross-sectional area of the moving object.

An empirical parameter called the Reynolds number determines which of the above two terms dominates:

$$\mathcal{R} = \frac{d\rho v}{\eta}$$

where for a simple sphere, the diameter $d(= 2r)$ is the obvious definition of the "size" of the particle[1]. For $\mathcal{R} < 1$, the viscous drag dominates (and thus there is no inertial coasting), while for $\mathcal{R} > 1000$ the viscous drag is completely negligible as compared to the inertial drag.

At higher velocities still, in compressible fluids one needs to account for the compression waves that get created (*e.g.* the sonic boom of a supersonic aircraft), and in incompressible fluids the surface layer of the fluid may lose contact with the rapidly moving object creating local vacuum (*e.g.* cavitation on the surface of the blades of naval propellers). Unusual fluids such as solutions of long polymers may experience flow in directions other than the direction of motion, as the entangled polymer molecules transmit the movement of the object to remote areas of the fluid and back. The fluid constrained into finite-size channels and tubes

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[1]For other shapes, an equivalent "hydrodynamic radius" r_h is used instead of r
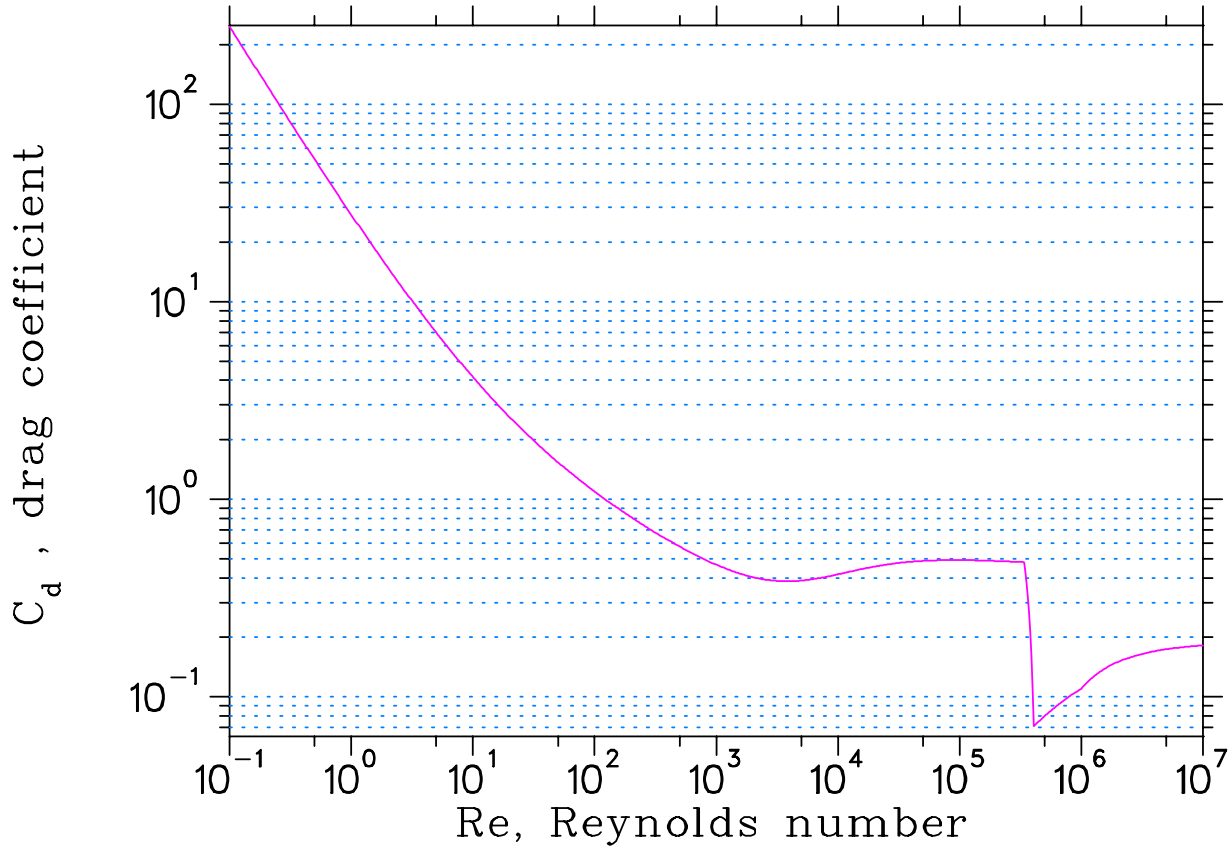
Figure 5.1: Drag coefficient on a sphere in an infinite fluid, as a function of the Reynolds number

also behaves differently from an infinitely-broad medium through which a small-size sphere is traveling. There are corrections that must account for all of these details.

A convenient way to express just such a correction is through an empirical factor called "the coefficient of drag", C_d, which can be measured experimentally and account for the details of shape, size, and the nature of the media through which an object is traveling. This dimensionless coefficient is well-known and its tabulated values can be found in handbooks. As a function of the Reynolds number, the experimentally measured values of the drag coefficient, C_d, constitute the so-called "standard drag curve" shown in Fig. 5.1. For a small sphere traveling through an infinitely broad fluid, this C_d provides an excellent agreement with the experimental drag force:

$$F_d = K_f \, C_d \, S \, \frac{\rho_0 v^2}{2}, \qquad C_d = C_d(\mathcal{R}) \tag{5.1}$$

The finite-size correction coefficient, K_f, describes how this equation is modified by the boundary conditions of a fluid confined in a finite-size vessel. For a sphere of diameter d traveling axially through a fluid channel (a pipe) of diameter D, a dimensionless ratio $x = d/D$ can be used to approximate this correction coefficient to within 6% via[2]

$$K_f = \frac{1}{1 - \alpha x^\alpha}, \qquad \alpha = 1.60 \text{ for } x \leq 0.6 , \tag{5.2}$$

in the range of Reynolds number values of $10^2 < \mathcal{R} < 10^5$.

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[2]R.Clift, J.R.Grace, and M.E.Weber. *Bubbles, Drops, and Particles*, p.226. Academic Press, 1978

For an object at rest, there is no drag force. As we apply a force to an object, it experiences an acceleration, its velocity grows, and with it grows the drag force that is directed opposite the velocity. This drag force counteracts the applied force and reduces the net force and the object's acceleration. Eventually, velocity increases to the point where the drag force is exactly matched to the applied force, bringing the net force to zero. From this point on, the object is in equilibrium, there is no acceleration, and the velocity remains constant, equal to its steady-state value called *the terminal velocity*, or v_t. For example, if an object is in a free fall through a fluid, this equilibrium condition corresponds to the weight of the object exactly equal to the viscous force:

$$F_d = W .$$

When the fluid in question is a gas such as air, the buoyant force F_b — equal to the weight of the displaced gas — is tiny and can be neglected. When the fluid is a more dense substance such as water, the above equation may need to be modified to include the buoyant correction due to the displacement of the fluid by the object

$$F_d = W - F_b = m_{\text{object}}g - m_{\text{fluid}}g = m_{\text{object}}g(1 - \rho_0/\rho) \tag{5.3}$$

where ρ is the density of the material of the object and ρ_0 is the density of the fluid. As you can see, only the ratio of the two densities enters into the expresiion since the volumes of the object and of the fluid it displaces are the same. For example, for an Al ball in water, $\rho_0 = 1.00 \times 10^3$ kg/m^3 and $\rho = 2.70 \times 10^3$ kg/m^3.

In this experiment you will explore this relationship by varying the force and measuring the terminal velocity of several spheres of different size.

Review questions

- What are the units of the viscosity coefficient η?

  . . . .

- What is the known value of η for water at room temperature? Cite the source of your value.

  . . . .

- In terms of its diameter d, what are the cross-sectional area S and the volume V of a sphere?

  $S =$ . . . .

  $V =$ . . . .

- For a sphere of 1-cm radius moving through water at 1 m/s, calculate the Reynolds number $\mathcal{R}$. What are the dimensions of $\mathcal{R}$?

  . . . .

- For the above sphere, which drag force (inertial or viscous) dominates? What is the expected coefficient of drag, C_d?

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CONGRATULATIONS! YOU ARE NOW READY TO PROCEED WITH THE EXPERIMENT!

Procedure and analysis

The experimental apparatus consists of two different-size metal spheres, a long cylindrical container filled with water, a pulley system with a weight platform and variable weights, an ultrasonic position sensor, a digital weight scale, a micrometer.
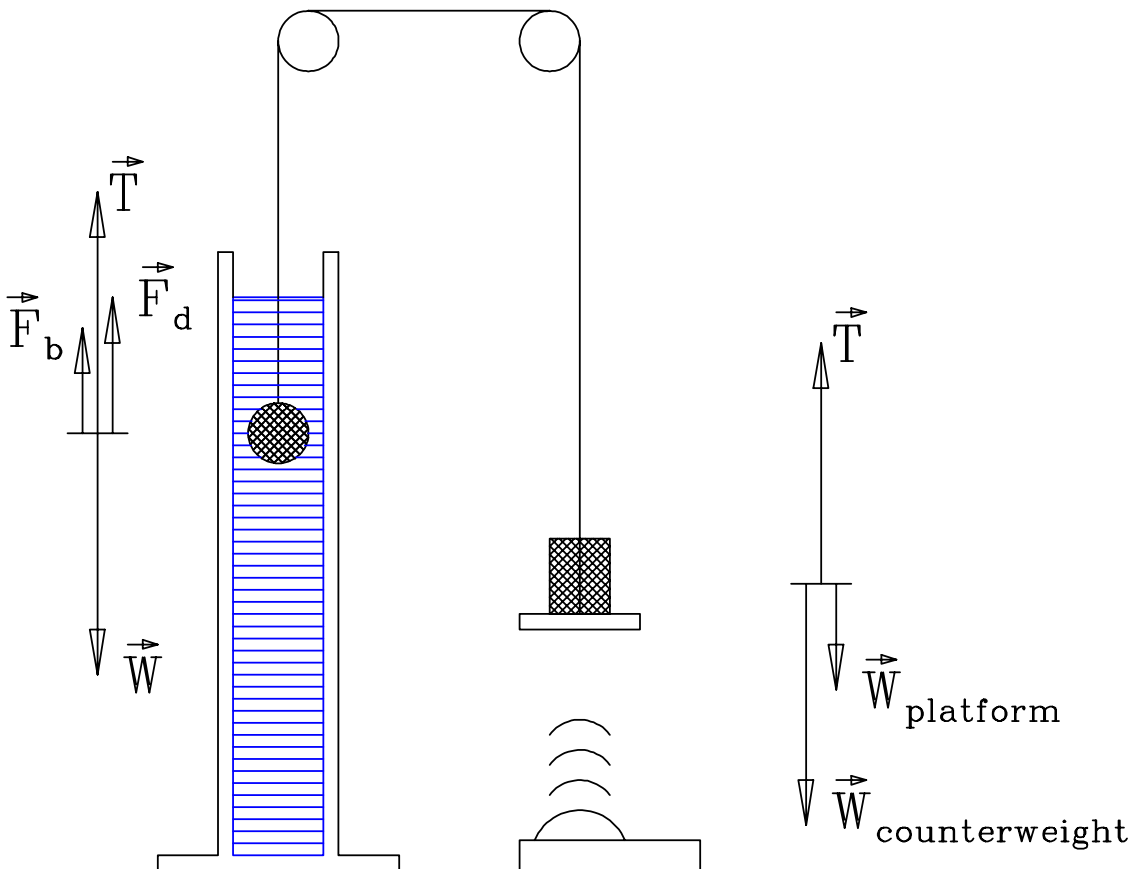

Figure 5.2: The apparatus for measuring the drag force of the fluid on a moving sphere

- Using a digital scale, weigh separately both spheres and the weight platform. You may also want to

verify the exact values of the various weights in the weight set.

$W_{\text{small}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $W_{\text{large}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

$W_{\text{platform}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $W_{\text{“10 g”}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

$W_{\text{“20 g”}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $W_{\text{“50 g”}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

- Using the micrometer, measure the diameter d of each sphere and calculate their cross-sectional area $S = \pi d^2/4$.

$d_{\text{small}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $d_{\text{large}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

$S_{\text{small}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$ $S_{\text{large}} = \ldots\ldots\ldots\ldots\ldots \pm \ldots\ldots\ldots\ldots$

- Measure and record the inner diameter D of the plastic cylinder filled with water. Verify that the cross-section is circular by measuring the diameter in a few different directions across the cylinder. Calculate the value of the parameter $x = d/D$ for both spheres.

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

- Select one of the spheres, adjust the length of the string so that the weight platform is at least 30 cm above the ultrasonic detector at its lowest position, and a few cm below the pulley at its highest position (*i.e.* when the sphere hits the bottom of the cylinder). Vary the weight you place on the weight platform (use single weights or their combinations), and measure a trace of position as a function of time as you let go of the string. Using *Physica Online* interface, each time gather about a hundred points spread out over the duration of the motion. For small weights (less that the weight of the sphere minus the buoyant force) the sphere will travel downward when released so start withe sphere at the top of the cylinder. For larger weights, the sphere will travel upward so start with the sphere at the bottom of the cylinder.

For each data set, identify the linear part of the trace, where the position-*vs*-time graph is a straight line, and fit only this range of data to a straight-line equation, `A*(x-B)*(x>`X_{min}`)*(x<`X_{max}`)` where X_{min} and X_{max} are two numbers selected by you from a visual examination of the plot. If the spheres never reach terminal velocity the position-*vs*-time graph will be curved for all times; skip such data sets. The small sphere may exhibit some oscillations due to the sphere wandering sideways on its way up or down; repeat these runs and select those with minimal wandering. Record all valid slopes (the parameter A of the fit) and their error estimates for several different combinations of counterweights. Note that the sign of the slope, and thus of the terminal velocity that you record, changes as the sphere switches directions of travel.

m, pull mass	v_t, small sphere	v_t, large sphere

- From the free-body diagram of Fig.5.2, drawn for the case of the sphere traveling downward as appropriate for the small pull mass of the counterweight, when $v = v_t = $ const,

$$+T - W_{\text{counterweight}} - W_{\text{platform}} = 0$$

and thus

$$F_d + F_b + W_{\text{counterweight}} + W_{\text{platform}} - W_{\text{sphere}} = 0\,.$$

Denoting the pull mass of the counterweight as m, the mass of the weight platform as m_p, the mass of the sphere as m_s, and using the expressions of Eqs. 5.1 and 5.3 we obtain

$$K_f\, C_d\, S\, \frac{\rho_0 v_t^2}{2} = m_s g \left(1 - \frac{\rho_0}{\rho}\right) - m_p g - mg$$

Dividing through by g we obtain

$$m = m_0 - \frac{K_f C_d S \rho_0}{2g}\, v_t^2\,,$$

where $m_0 = m_s(1 - \rho_0/\rho) - m_p$. For each of the spheres, record the pull mass used, m, as a function of the measured terminal velocity, v_t.

- Repeat for the sphere traveling upward; you need to re-draw the free-body diagram of Fig.5.2 as appropriate for this case. Remember that the drag force is always opposite to the velocity of the object moving through the fluid.

- Using the data obtained for both direction of travel, enter the values obtained in the previous steps into *Physica Online* and plot m vs. v_t. Fit to a quadratic expression, `A+B*sign(x*x,x)` (the function `sign()` here accounts for a change in the direction of the drag force as v_t changes sign).

  Report the results for both spheres. Correct for the finite size of the water pipe (K_F will be different for the two spheres), and see if the drag coefficient you measure is the same in both cases. Compare the calculated C_d with the standard drag curve of Fig. 5.1 and comment on any discrepancies.

	B$\pm\Delta$B	$K_F \pm 6\%$	$C_d \pm \Delta C_d$
small sphere			
large sphere			
average	—	—	

IMPORTANT: BEFORE LEAVING THE LAB, HAVE A T.A. INITIAL YOUR WORKBOOK!

Discussion

To complete this lab, submit a typed or neatly written Discussion of the results of the experiment (double-spaced, 400 to 600 words). Attach your computer printouts and worksheets to this Discussion. Discuss the following issues as part of your Discussion; as always this is not a complete list:

- What is the physical meaning of the fit parameter `A` ?

- From the shape of the plot of m *vs.* v_t, is there any evidence of an additional force due to the friction in the pulleys? How would you estimate this force?

- How does your value of C_d compare to the known values? What are the likely reasons for the discrepancy, if any?